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AUTHOR Glaus, Marlene, Ed.; Myers, Donald, Ed.  
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## ABSTRACT

The Minnesota School Mathematics and Science Teaching (MINNEMAST) Project is characterized by its emphasis on the coordination of mathematics and science in the elementary school curriculum. Units are planned to provide children with activities in which they learn various concepts from both subject areas. Each subject is used to support and reinforce the other where appropriate, with common techniques and concepts being sought and exploited. Content is presented in story fashion. The stories serve to introduce concepts and lead to activities. Imbedded in the pictures that accompany the stories are examples of the concepts presented. This unit extends the work on symmetry presented in an earlier unit. As well as mirror reflections and translatory motion, symmetry about a point is developed. Activities focus on finding and creating patterns. In the process children are led to discover a number of geometric relationships. Worksheets and commentaries to the teacher are provided and additional activities are suggested. (JP)

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**UNIT XIV**  
**SYMMETRY**

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MINNESOTA SCHOOL MATHEMATICS AND SCIENCE CENTER

1965



MATHEMATICS  
FOR THE  
ELEMENTARY SCHOOL

UNIT XIV

Symmetry

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# MINNESOTA MATHEMATICS AND SCIENCE TEACHING PROJECT

JAMES H. WERNTZ, JR.  
Associate Professor of Physics  
University of Minnesota  
Project Director

PAUL C. ROSENBLOOM  
Professor of Mathematics  
Teachers College, Columbia University  
Mathematics Director

## Symmetry

MARLENE GLAUS  
Third Grade Teacher  
Richfield, Minnesota

UNIT EDITOR and STORY AUTHOR

DONALD MYERS  
Associate Professor of Mathematics  
University of Arizona

CONTENT EDITOR

ARTHUR MAUD  
Director, Project IV: Music School  
Minneapolis

MUSIC

DAVID RATNER  
Assistant Professor of Art  
Boston University

ARTIST

ELLEN RATNER  
Natick, Massachusetts

ARTIST

JUDITH HORAZDOVSKY  
Undergraduate Assistant  
University of Minnesota

**We are deeply indebted to the many teachers who  
used earlier versions of this material  
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## **MATERIALS**

1. 1 large class mirror (rectangular or square size)
2. 1 can powder tempera paint (any color)
3. 1 heavyweight paint brush
4. white drawing paper (1 sheet per child)
5. tracing paper (5 sheets per child)
6. crayons (1 box per child)
7. pencils (1 per child)
8. rulers (1 per child)
9. mirrors about 4 x 6 inches in size  
(1 per child, glass or metal mirrors)
10. scissor (1 per child)



## INTRODUCTION

In this unit the children will study a fundamental geometric concept of rigid motion, that is, a motion with distances unchanged. Two types of simple motion are presented to help the children to find what patterns are unchanged by these motions. In the process children will discover a number of geometric relationships.

The study of symmetry is important not only in mathematics but also in art, biology and physics.

It is to be noted that in this unit the dotted lines in designs on the various pages indicate the lines of symmetry which will be more defined in each specific activity. The pattern to the left of the line of symmetry becomes a mirror reflection of what is drawn on the right side of the line. The same is true of the left side reflecting to the right side.

Reference is given to the following books for a more extensive definition and appreciation of the meaning of symmetry:

Celebonovic, Stevan, The Living Rocks, Philosophical Library, 1957.

Feininger, Andreas, The Anatomy of Nature, Crown Publishers, 1956.

Jirovec, O., Boucek, B., and Fiala, J., Life under the Microscope, Spring Books, London, 1959.

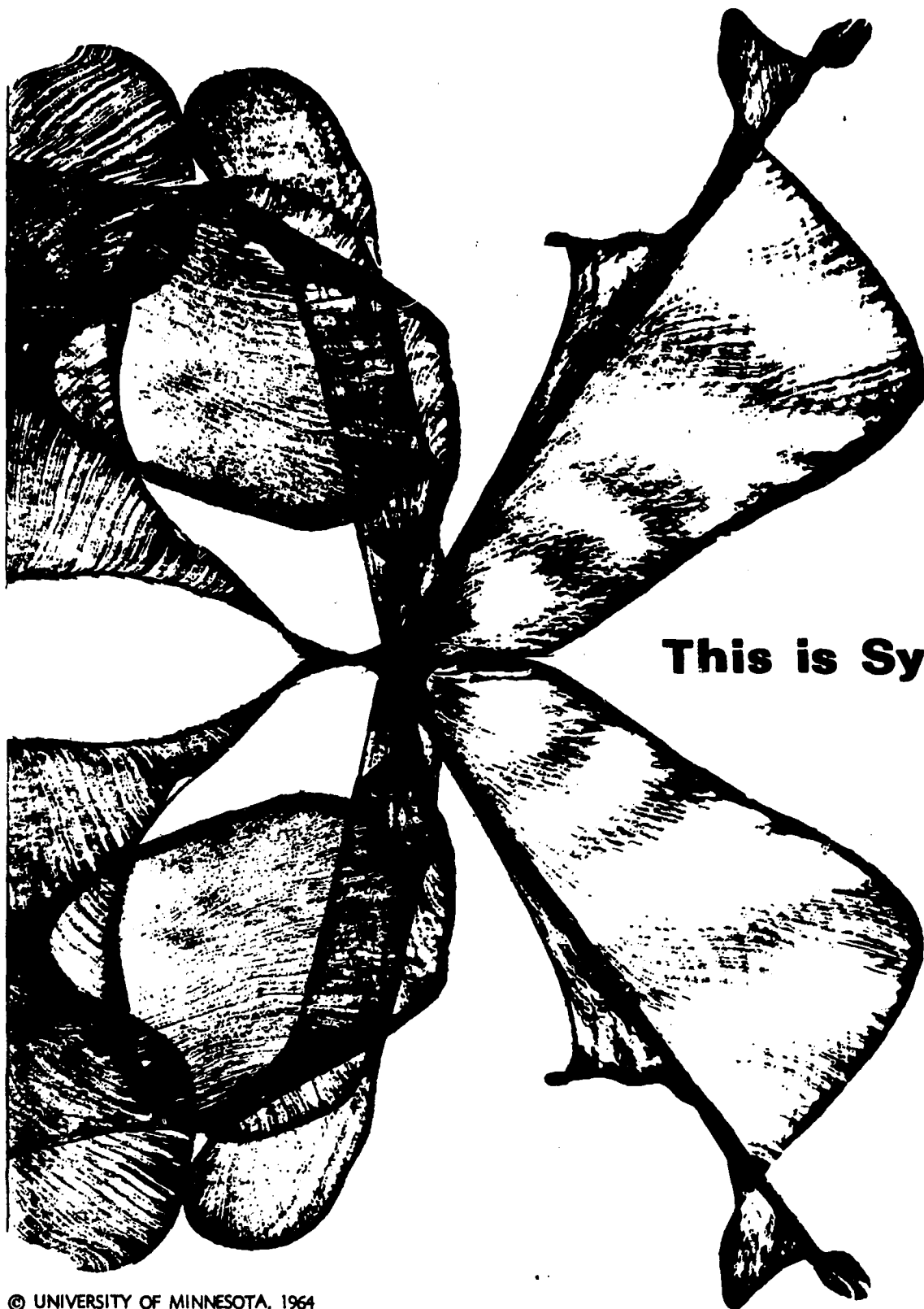
Strache, Wolf, Forms and Patterns in Nature, Crown Publishers, 1956.

Thompson, D'Arcy, Growth and Form, Cambridge University Press, 1952.

Weyl, Herman, Symmetry, Princeton University, 1952.

If kaleidoscopes are made available, they will show symmetrical designs of balance which can often be changed by a simple rotation of the instrument.

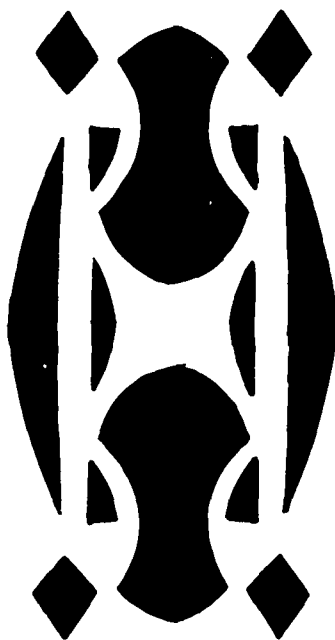
The story titled, "This is Symmetry" and the various activities that follow will help children be more aware of symmetrical designs in and out of nature.



**This is Symmetry**

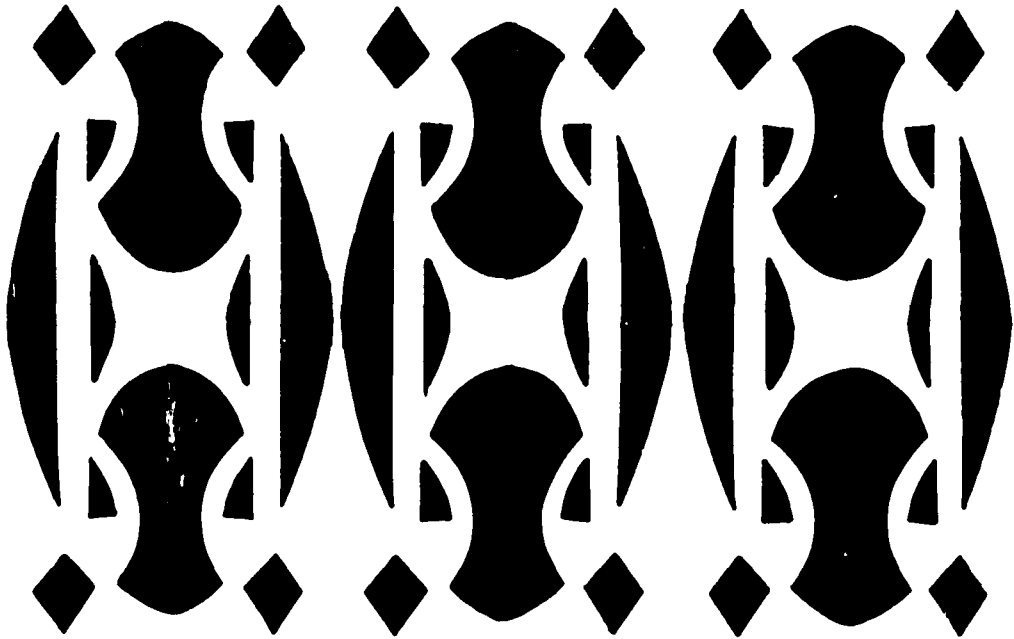
© UNIVERSITY OF MINNESOTA, 1964

## THIS IS SYMMETRY



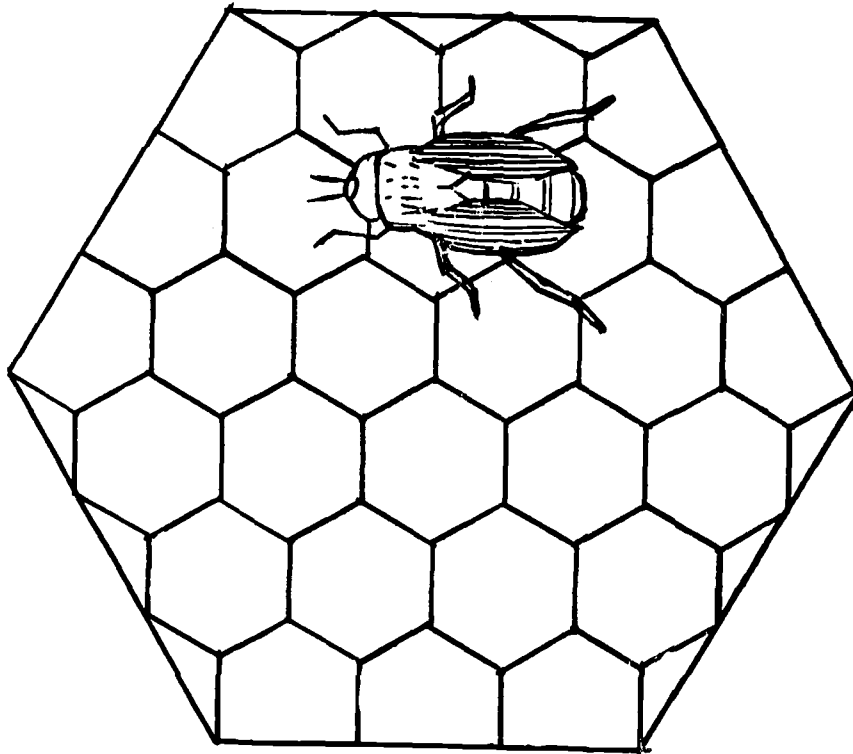
Symmetry is a pattern. It may be a simple pattern or one with many parts. The parts of the pattern must be in balance with the other parts.

## THIS IS SYMMETRY



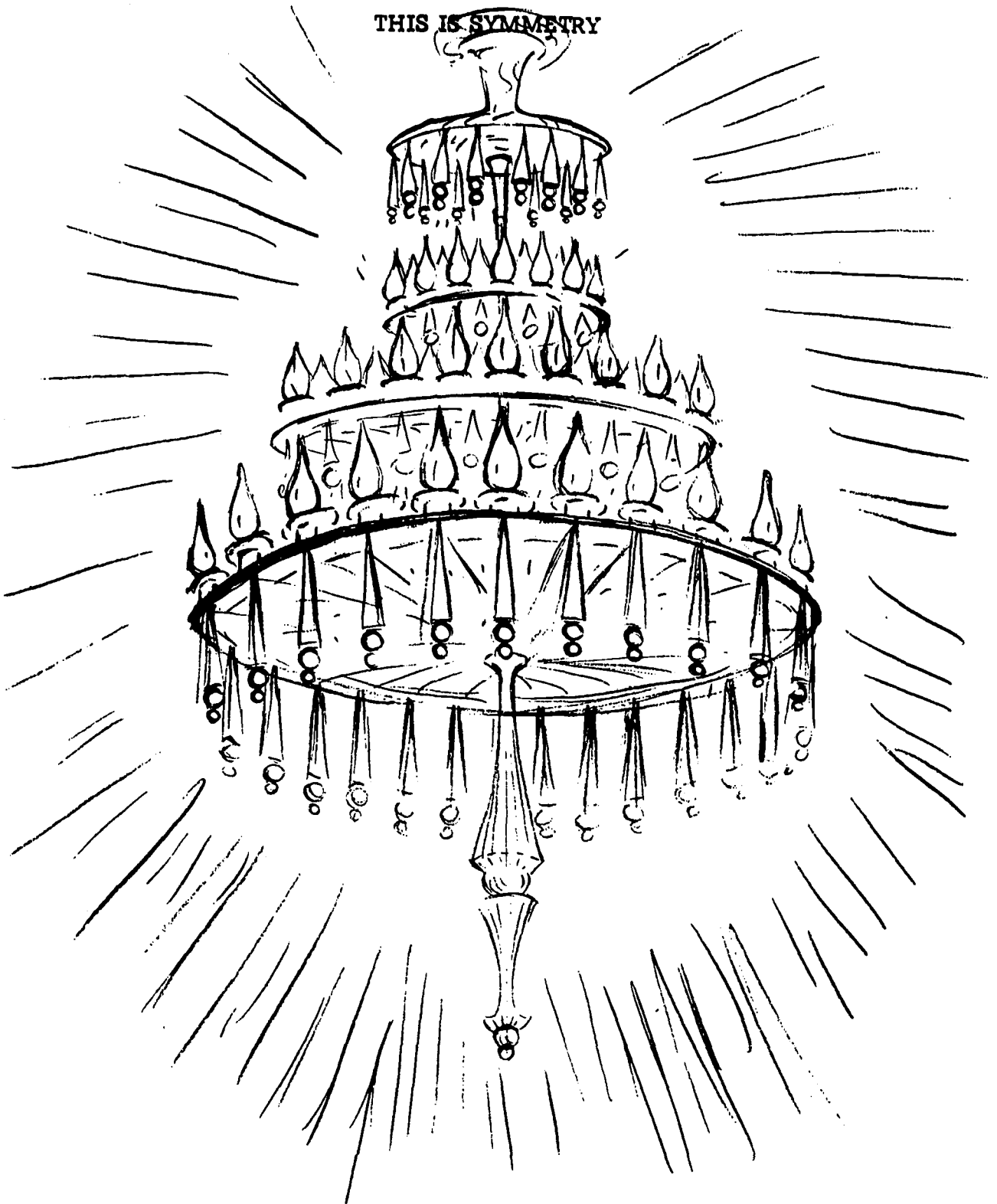
The pattern may be repeated once or twice or more times than that. It may be a repeated pattern in a design like the one in this picture or it may be the repeating pattern of a fence around someone's yard or it could be the arrangement of the windows in a sky scraper. .

## THIS IS SYMMETRY



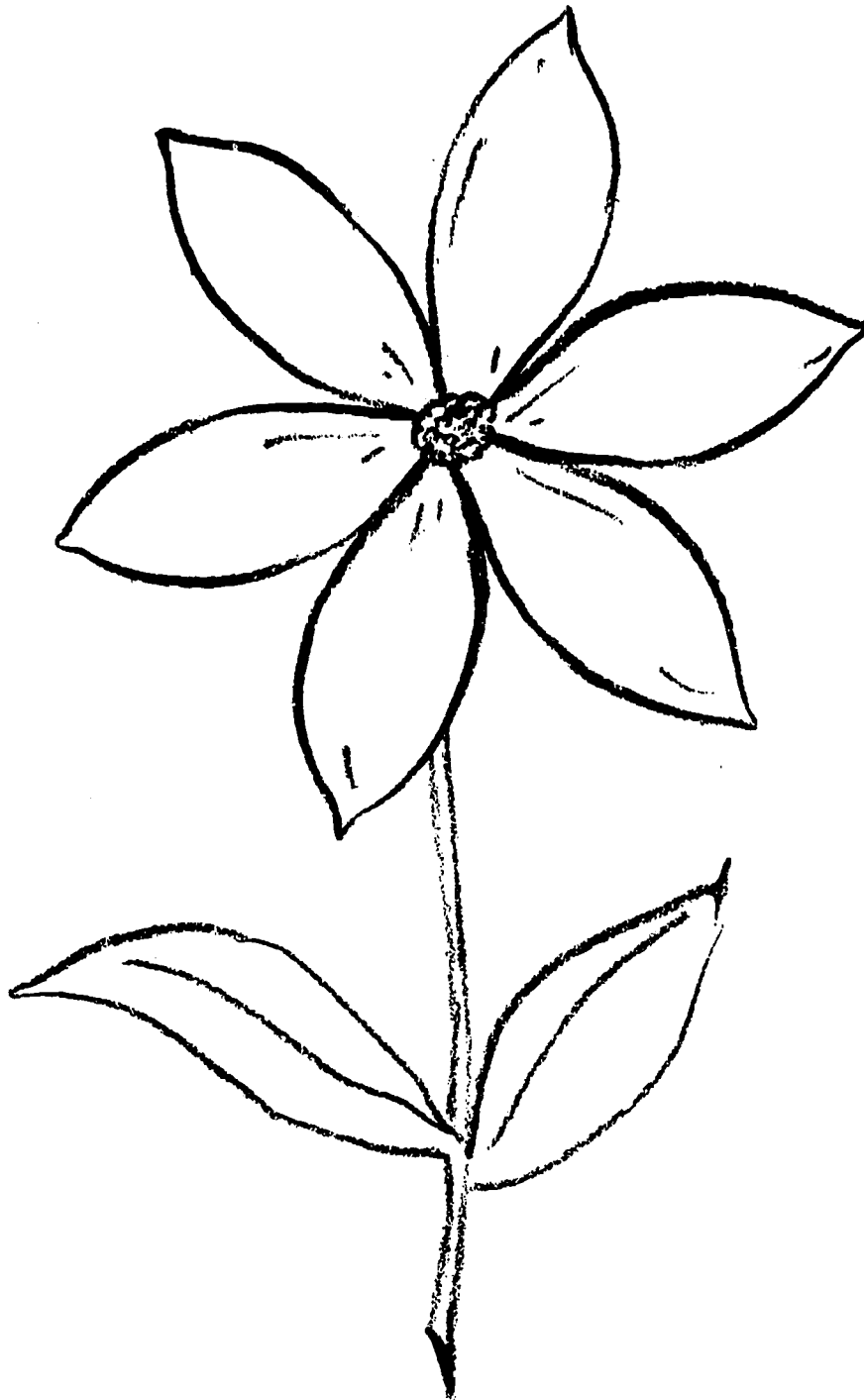
Symmetry is seen in the honeycomb of a bee. Each cell in the hive has six sides. The cells repeat each other.

THIS IS SYMMETRY



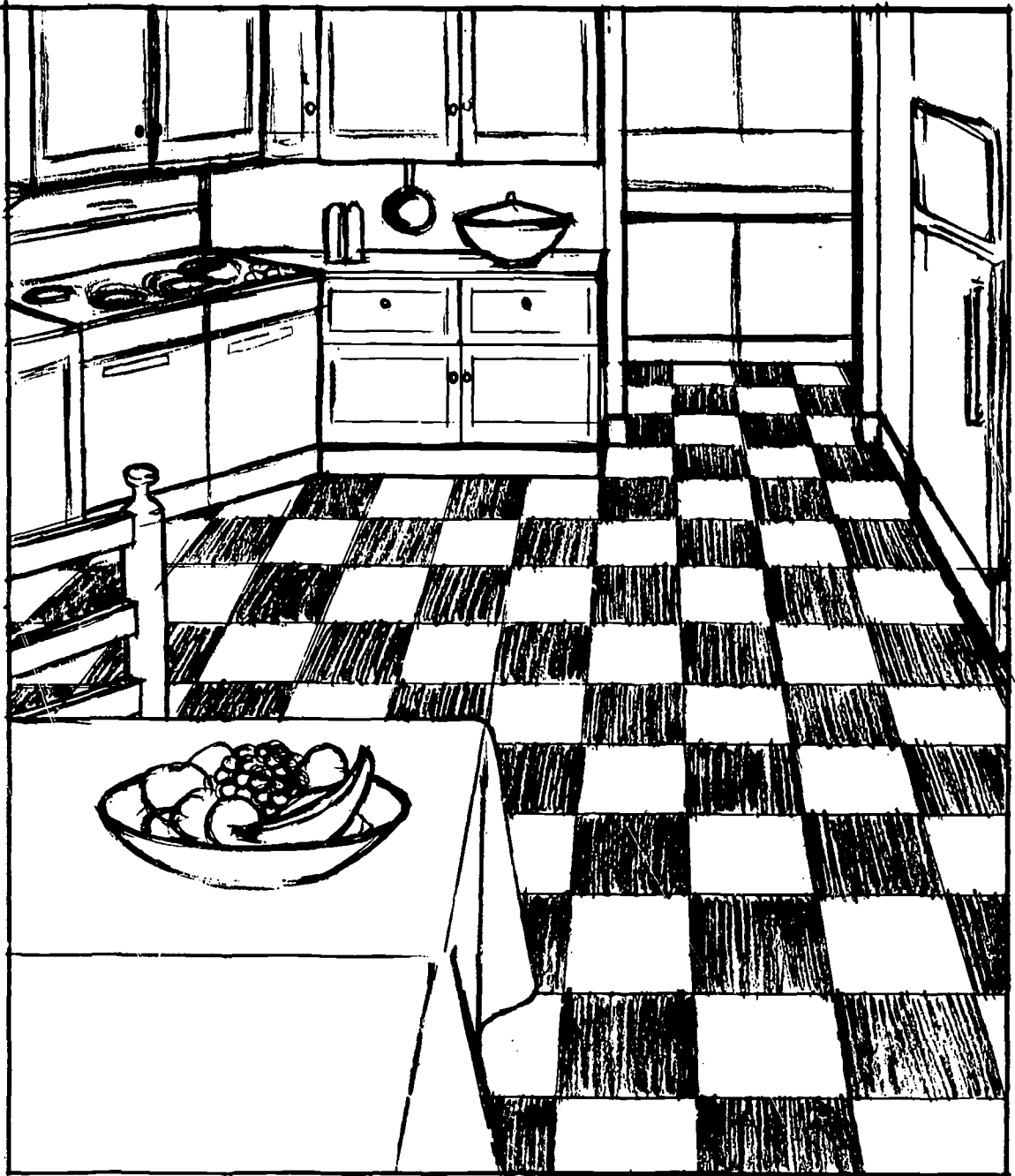
We can see symmetry in a chandelier of lights. Each light helps the others to be a design of symmetry. The lights repeat their order round and round.

## THIS IS SYMMETRY



A first flower blooms in the warmth of the springtime. It unfolds six colored petals. Each petal has about the same length and shape and size. One petal is almost in balance with the other petals.

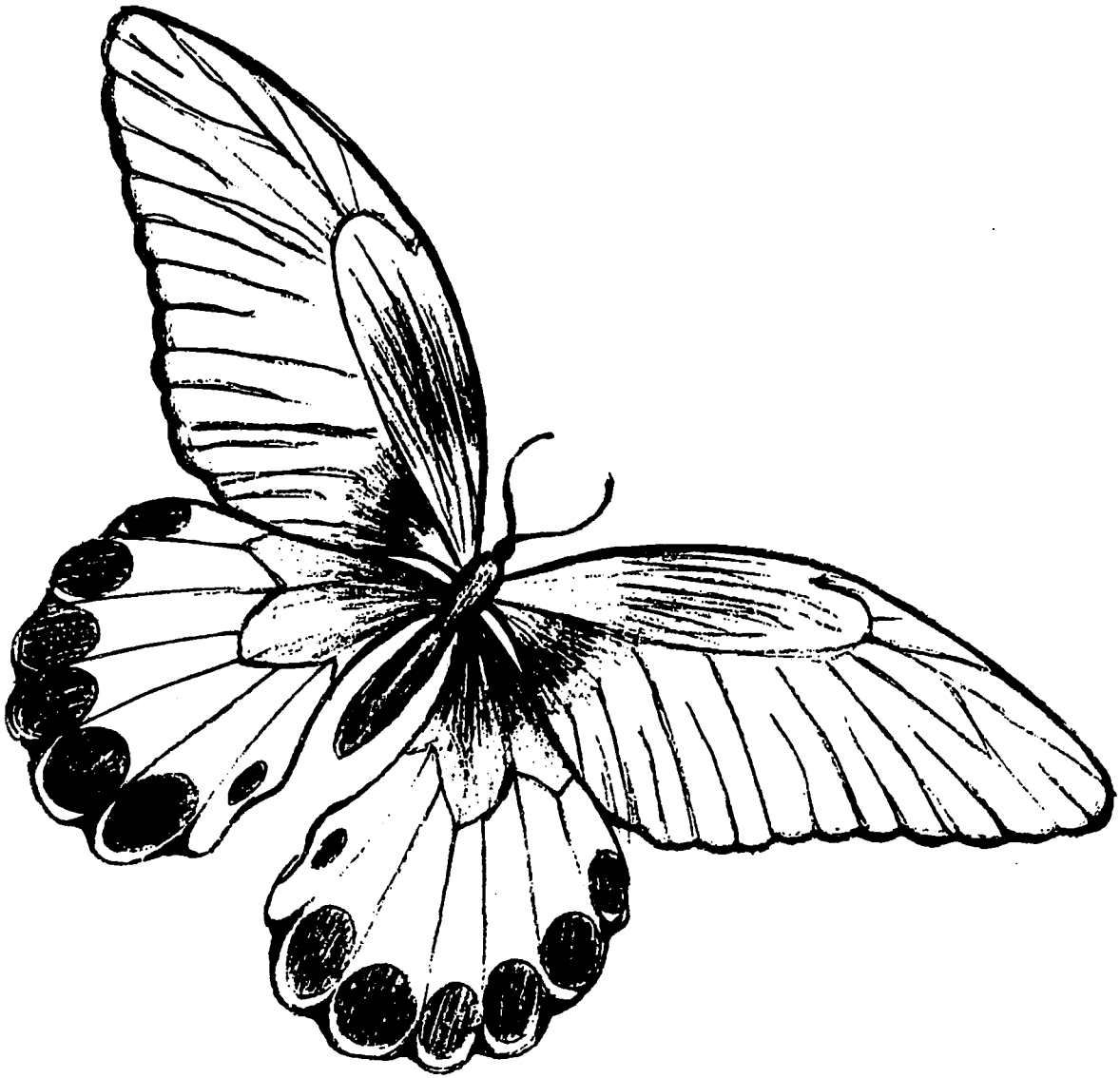
## THIS IS SYMMETRY



Some kitchen floors are covered with square tile pieces. One tile is black, the second is white, then the pattern starts all over again.



## THIS IS SYMMETRY



Wherever symmetry may be, it helps the world to be  
a more beautiful place.

## THIS IS SYMMETRY



There are thousands and thousands of patterns of symmetry. Only a few of the patterns are perfect. Can you find some right now?

### Background for the teacher

The following are some other patterns of symmetry: (If children are given the opportunity to name patterns which they are familiar with, it is quite possible that these and many others will be mentioned.)

#### Man-made symmetry

1. stair-steps leading from one floor to another
2. a long ladder standing next to a tree
3. a border going round and round the top of a wall-papered room
4. the arrangement of cookies on a tray
5. registers and radiators in the rooms of some buildings
6. oil well towers in big open fields
7. the weaves of straw in baskets and purses
8. the spans of iron bridges
9. designs in clothing and wall-paper

#### Symmetry in nature

1. the sections of a tall bamboo plant
2. a delicate white snowflake in a month of winter
3. the look of rings in the stump of a tree
4. the inside sections of citrus fruits
5. the flaky scales of a fish
6. the long, colored feathers of a strutting peacock
7. the seeds growing close together in the center of a yellow sunflower

It is to be mentioned that most examples in the story and those listed on this page are not examples of perfect symmetry, but they will serve to help children develop a closer awareness of symmetrical patterns which are displayed for them to see in their natural environment.

### Activity

Making a bulletin board displaying symmetrical designs and arrangements

### Materials needed

A collection of materials contributed by the children

### Background for the teacher

In stimulating a better awareness of symmetrical designs, ask the children to make a collection of symmetrical designs which they might have at home. They may be encouraged to look for such examples of designs in:

1. magazine pictures
2. scraps of materials
3. wall-paper pieces

Nice size examples for display on a bulletin board should be discussed to prevent the sizes from being too large or small.

## BILATERAL SYMMETRY

### Completing a Pattern

#### Commentary for Worksheet 1

Observing corresponding points in a symmetrical design

#### Materials needed

Provide for each child:

crayons

pencil

#### Vocabulary

Introduce these words:

symmetrical

corresponding points

vertical line

horizontal line

diagonal line

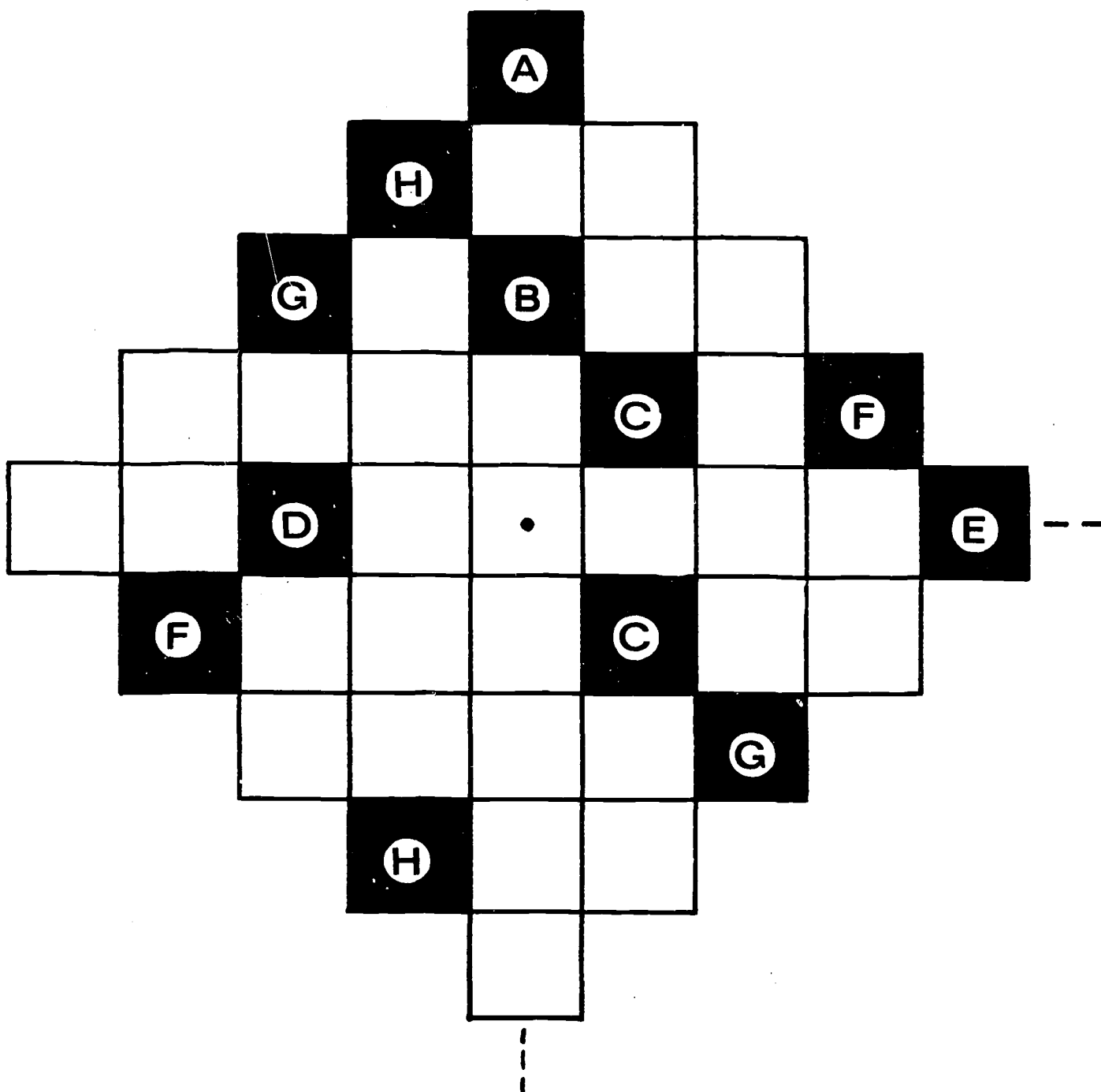
#### Background for the teacher

Ask the children to see that every point has a corresponding point in the design. Encourage them to always look for their boxes in a horizontal location with the exception of boxes A and B which are located vertically.

Ask them to locate point A. Have them put an A in the corresponding box which falls vertically. Continue with all other letters in sequence, each time labeling all the corresponding square regions (A through H). Have them shade the square regions in around their letters so they will see what design is formed. (It will resemble a checkerboard design where one square is light and the next dark.)

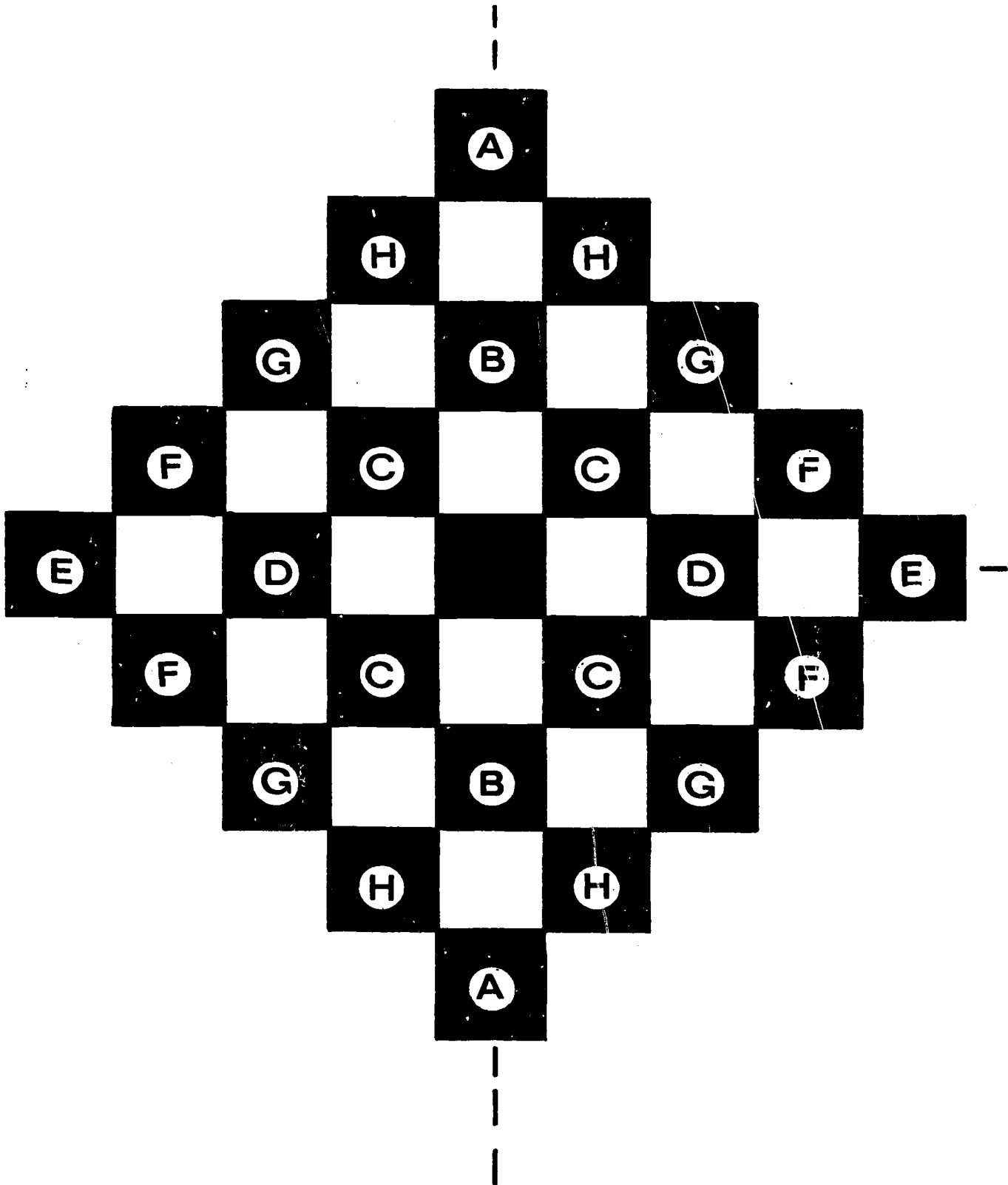
Have the children fold the paper in half either vertically or horizontally over the dotted lines (which indicate the lines of symmetry) and ask them to hold their papers towards the light. If folded and shaded carefully, they will be able to observe that each quarter part is in balance with each other part according to corresponding points of lightness and darkness.

The children should realize that the line of symmetry can have any orientation, horizontal, vertical, or oblique. For better understanding of the class, show examples using these lines.



Color the center box. Find the boxes that your teacher asks you to find. Put corresponding letters in them. Then shade in the corresponding lettered boxes with your pencil. What design do you see?

Answers for Worksheet 1



## Reading a Mirror Message

### Commentary for Worksheet 2

Letting children discover how a word message that is written backwards can appear to read correctly in a regular left to right sequence

### Materials needed

Provide for each child:

mirror

sheet of tracing paper

### Background for the teacher

Instruct the children to look at the message written backwards on their worksheets. Ask them to think of two ways in which they could read the message with ease and understanding.

Solutions:

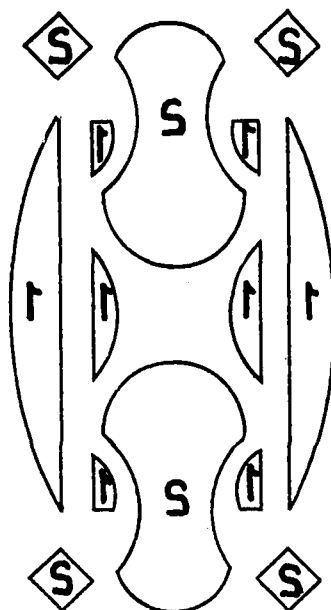
1. The message can be read with the aid of a mirror which should be placed along the left side of the message.
2. The message can be read by viewing the back side of the paper held up to a light.

It is suggested that the children have an opportunity to write messages which appear to read backwards. This can be done by having them write a sentence or two in the usual way, hold it up and turn it around to the light of a window and trace over it. It will then appear to read backwards.

The children might want to share their messages with other children in the class.



\_\_\_\_\_ Name



## Color This Pattern

Color the parts that say **1** orange.

Color the parts that say **2** black.

Put some more parts on the pattern.  
Be sure they balance with each other.

## A Symmetrical Face

### Commentary for Worksheet 3

Observing and recording a mirror reflection

### Materials needed

Provide for each child:

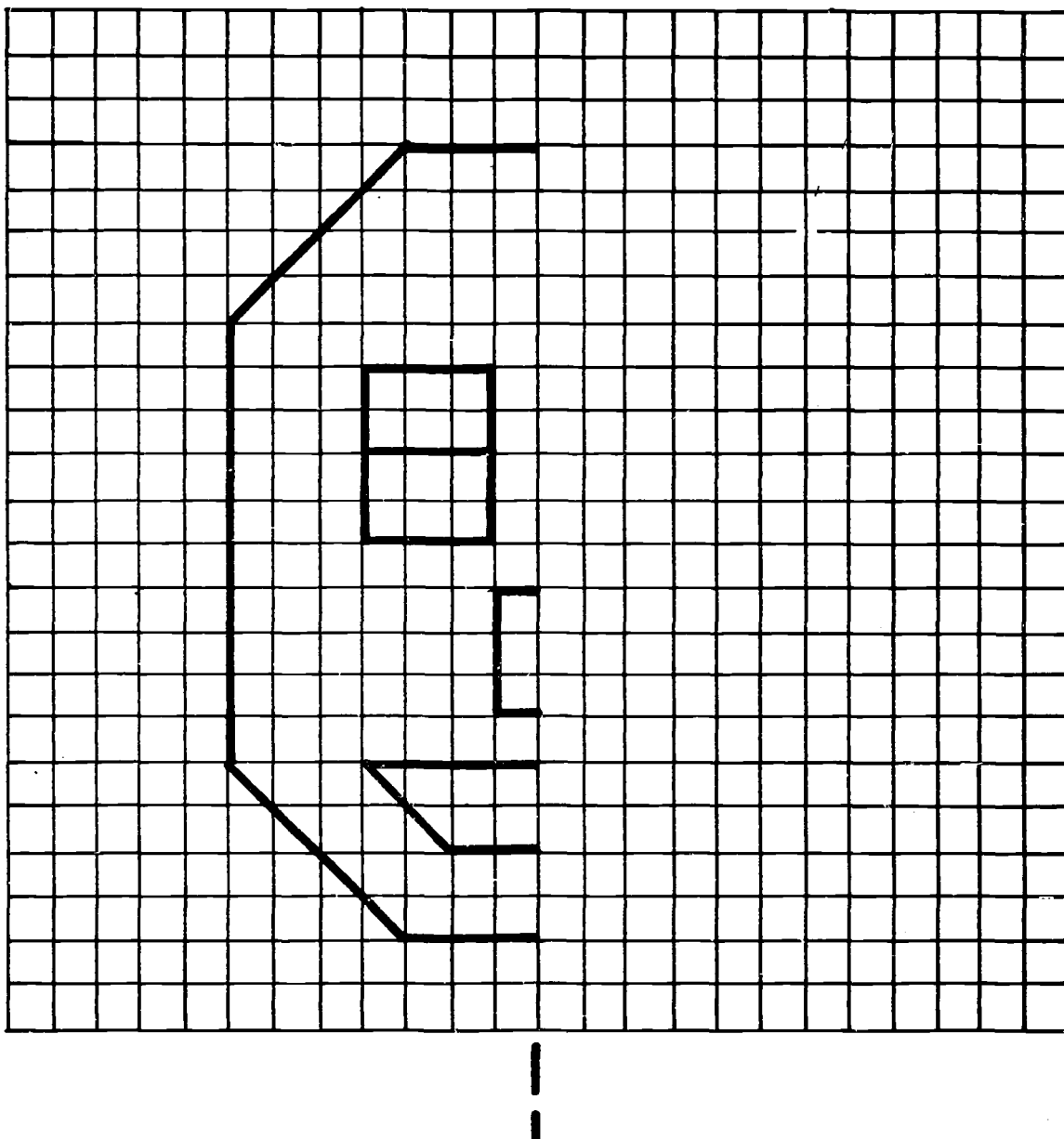
black crayon

mirror

### Background for the teacher

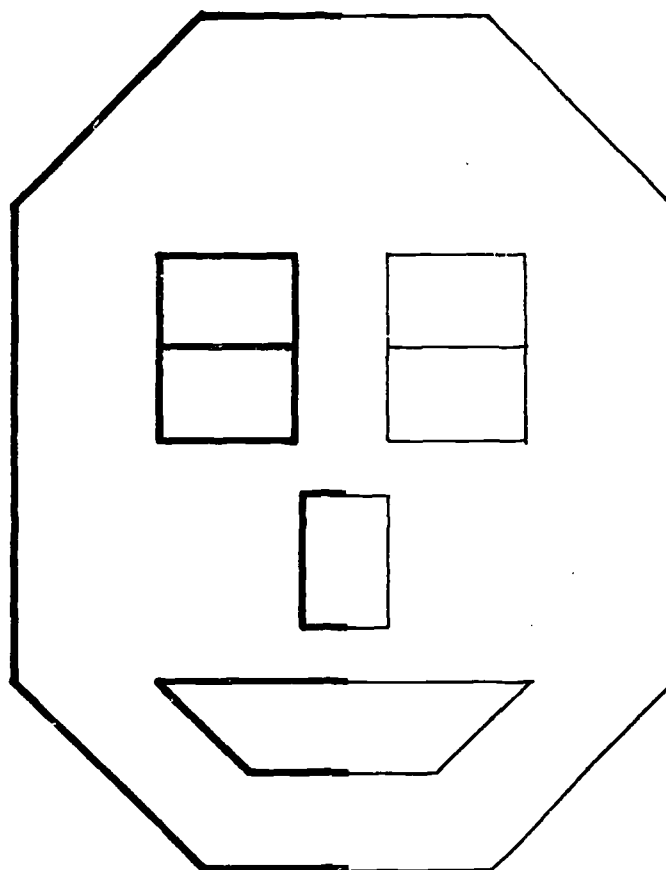
Ask the children to hold their individual mirror along the dotted line. Have the class record their mirror reflections to complete the robot-looking face on the right side of the line of symmetry with a black crayon.

# Worksheet 3



Hold your mirror along the line of symmetry.  
Use your ruler to help you draw the right side  
of the face as you saw it in the mirror.

Answers for Worksheet 3



## A Mirror Reflection

### Commentary for Worksheets 4 and 5

Discovering a mirror reflection used in relation to symmetry

#### Materials needed

One large class mirror

Provide for each child:

pencil

ruler

mirror

pair of scissors

#### Vocabulary

Review the term mirror reflection which was originally introduced in the first grade symmetry unit.

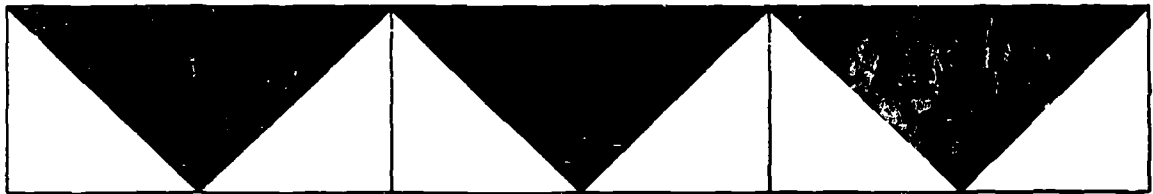
#### Background for the teacher

Allow time for each child to cut off the top of his paper corresponding to the top edge of his design. Direct the children to hold the illustrated border with the top edge against a large mirror.

After they have discovered the new double looking design, have them complete the picture which they saw in the mirror on the following exercise which is half completed.

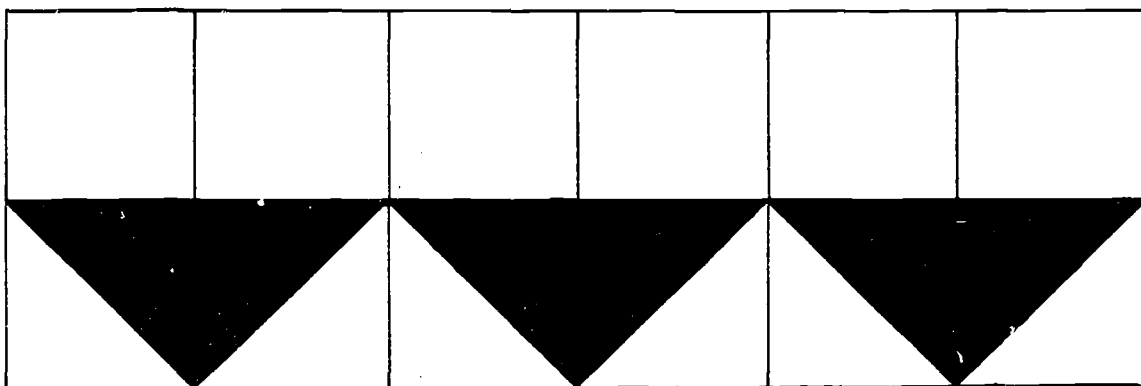
If a larger class mirror is available, it would provide a clearer picture for children to see as they come to it one by one with their papers.

# Worksheet 4



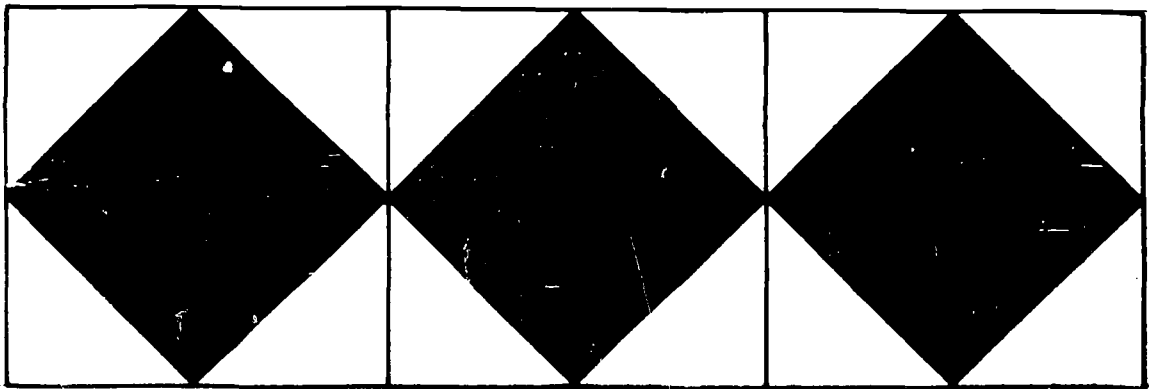
Cut off the paper above the border. Hold it  
against a mirror. What do you see in the mirror?

# Worksheet 5



Draw in the mirror reflection on the top half of this pattern.

Answers for Worksheet 5





## Making a Border

### Commentary for Worksheet 6

Introducing a border

### Materials needed

Provide for each child:

- mirror
- pencil
- sheet of white paper

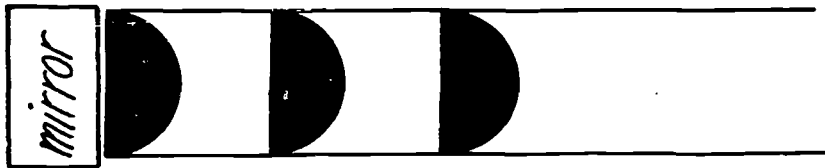
### Background for the teacher

Have the children cover the border number two with a sheet of white paper so only the top border is visible.

Explain to them that this picture is a border (a pattern repeated over and over again in sequence).

Have the children identify the outline of the semi-circle, and allow time for them to talk about the border. Also give them time to name places where they have seen borders.

Make a drawing on the board for an example.



Place a mirror next to the left edge of the border. Don't let the children see the mirror image, or use their mirrors, but have them think about what pattern would be reflected in the mirror if they could see it. Have them draw their thoughts in the white spaces on their border number one.

Now instruct the children to place their mirrors next to the left edge of the border number two and then continue by repeating the placement of the mirror along the flat edge of each semi-circular region. They will discover that the mirror reflection of the semi-circular region makes a complete circular region each time.

With the help of the mirror, have the children complete filling in border number two.

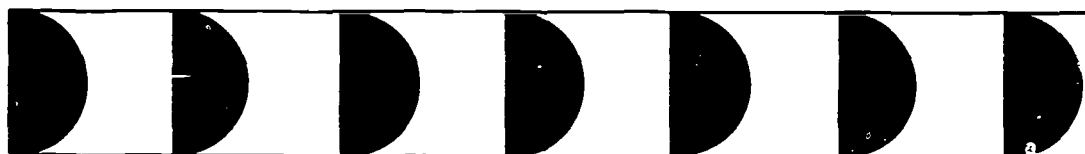
When the children have completed border two, have them compare it with their border number one to see if they had pictured correctly the mirror reflection.

## Worksheet 6

Border 1



Border 2



Do not use your mirror in border one.

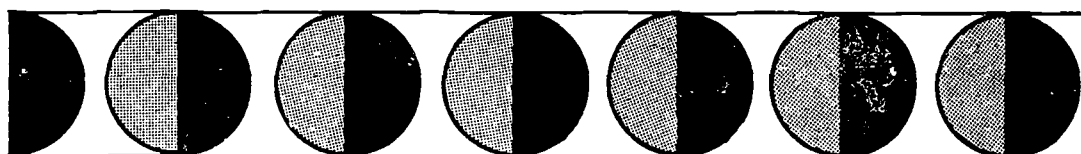
Do what your teacher asks you to do with your  
mirror in border two.

# Answers for Worksheet 6

Border 1



Border 2



## A Semi-circle Mirror Design

### Commentary for Worksheet 7

Making a border having a horizontal line of symmetry

### Materials needed

Provide for each child:

pencil

mirror

### Background for the teacher

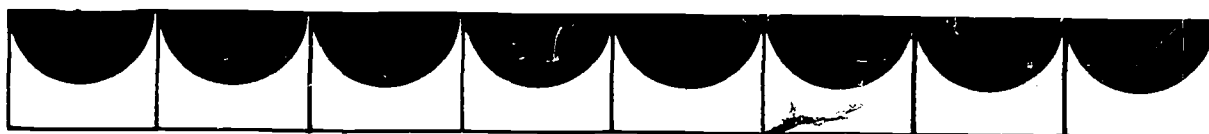
The children should be directed to do exactly as they did in the previous exercise in filling in their border.

They should place their mirror on the top edge of border number one and then complete filling in border number two.

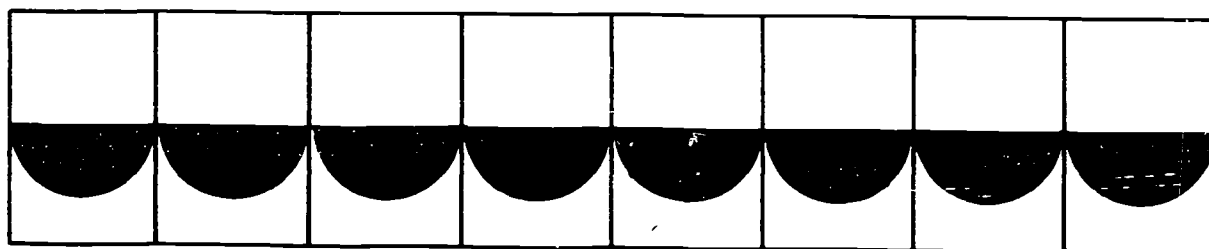
Give them the opportunity to use their mirrors as much as they wish in order to complete the mirror reflection.

# Worksheet 7

Border 1



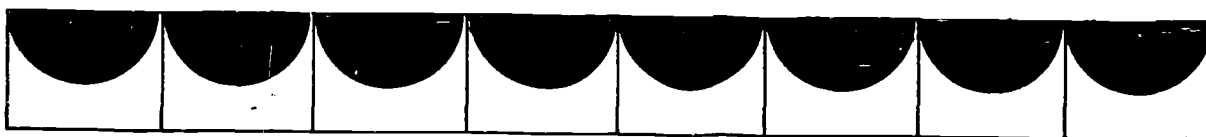
Border 2



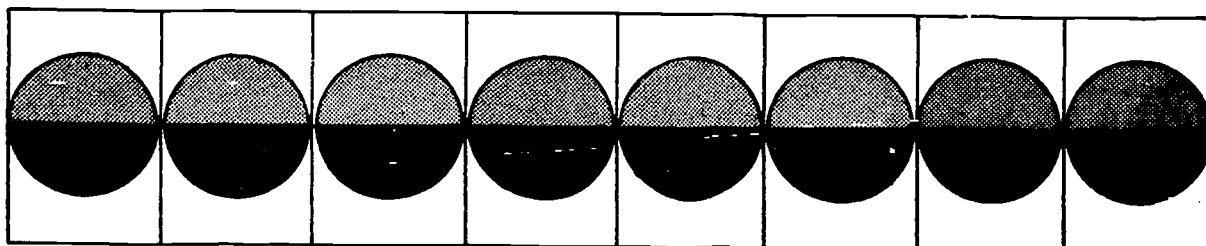
Hold your mirror along the top of border one.  
Draw what you saw in border number two.

Answers for Worksheet 7

Border 1



Border 2



## A Counter Placed with Counters

### Commentary for Worksheet 8

Estimating spatial relationships  
Observing a symmetrical pattern

### Materials needed

Seven cardboard counters

### Background for the teacher

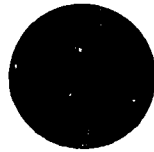
Place a circular cardboard counter on a table. Have a few small groups of children view the counter at a time. Have them discover how many counters they think would fit together side by side around the edge of the counter.

If each child has counters, they can try to see if they estimated correctly.

They may want to estimate larger circle sizes such as the number of quarters that will fit around one quarter, etc.

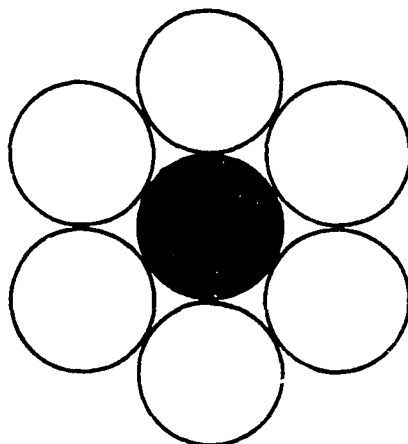


## Worksheet 8



This is a picture of a penny size circle. How many penny size circles do you think could fit around it side by side?

## Answers for Worksheet 8



## Counters on a Paper

### Commentary for Worksheets 9 and 10

Forming designs in relation to symmetry  
Learning the strategy of winning a game

#### Materials needed

Provide for every two children:  
twenty-four penny-size cut-outs

#### Background for the teacher

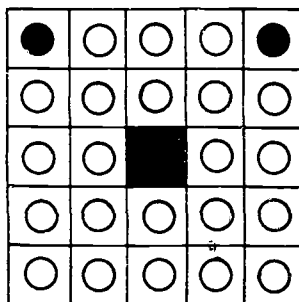
On the child's sheet are twenty-four penny-size circles to be cut out. They are to be used only if there are not penny-size counters available.

On the second sheet is the circle game which is not to be cut out. This is a game where each child has a partner. The partners need only twenty-four penny-size circles or counters for the two of them.

The game has a blacked in area in the center which should be omitted from any plays.

The partners decide who should begin by making the first move. The first player covers a spot on the game with his counter or cut-out anywhere that he chooses. The second player must place his counter on a spot exactly opposite on the game always moving in a horizontal position with the exception of vertical moves through the center. This movement of plays in horizontal and vertical ways should be discussed before hand.

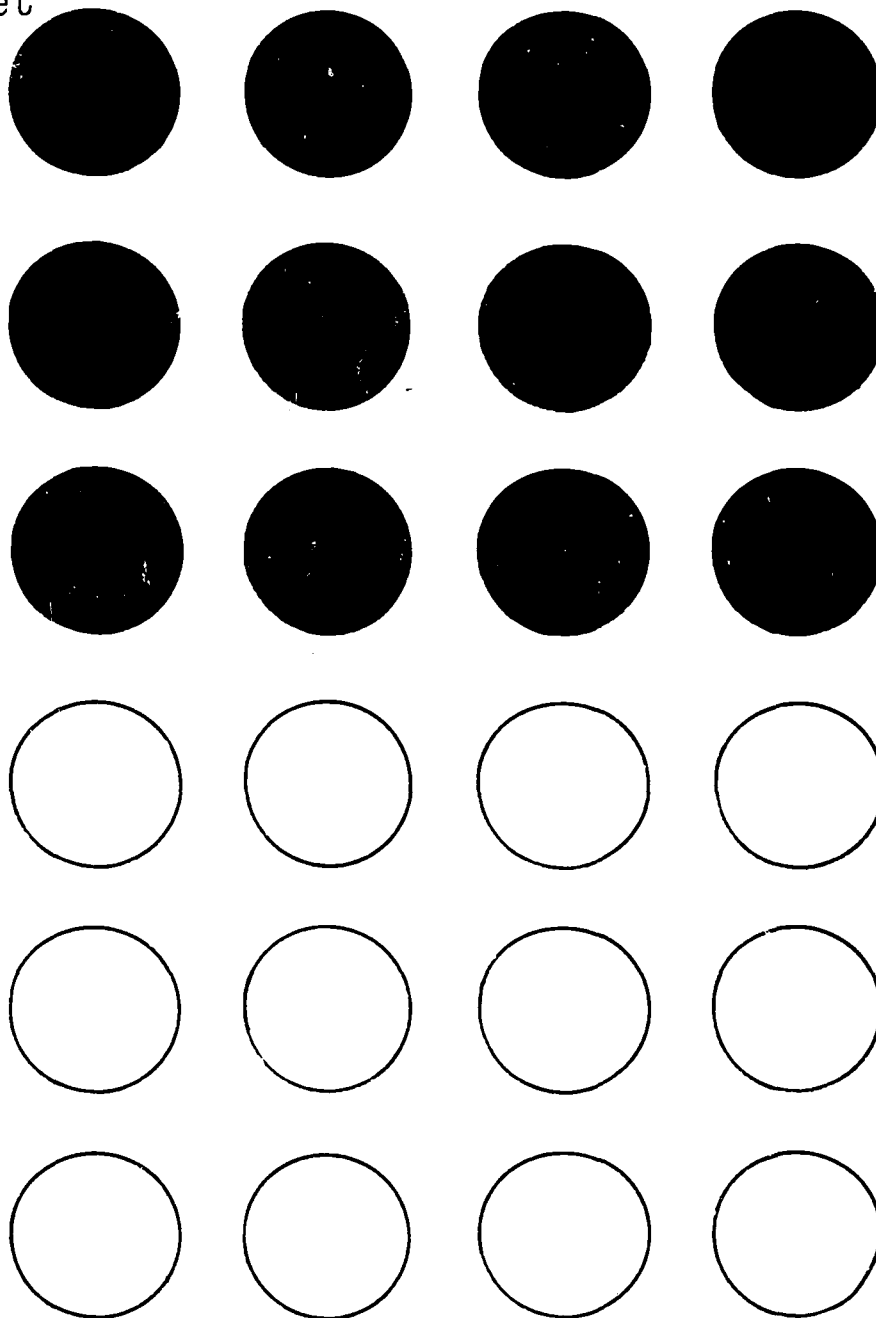
#### Example



The first player will always have the fun of moving where he wishes. Player 2 becomes a follower because he always has an exact spot to move on which is the corresponding point played on by player 1. Player 2, however, has the good fortune of becoming the winner each time because the last player to move is always the winner of this game.

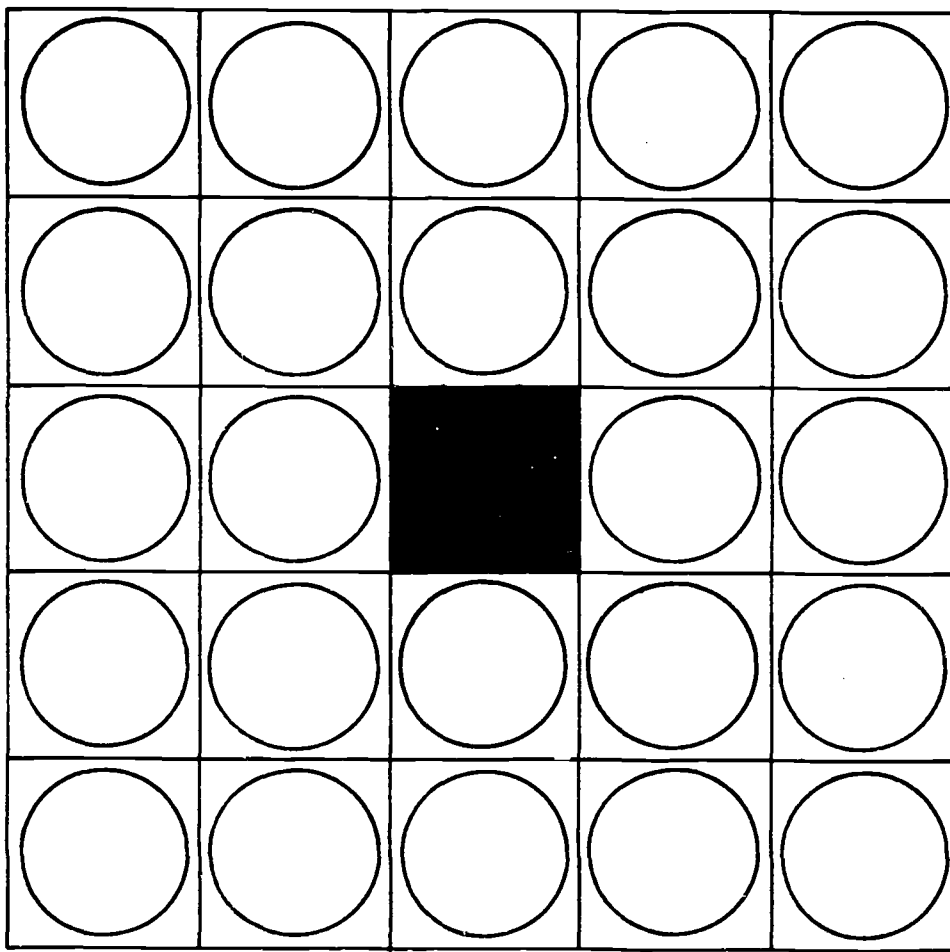
Let the children play until they discover by themselves how to win the game. Be sure that the individual couples of children tell the teacher privately while others try to solve the solution.

Some children might need more than one sitting to solve the winning strategy of the game.



Cut out these circles. One player should use the black ones. It is suggested that the other player use the white ones so as to differentiate between the players' moves. If counters are available, this page may be omitted.

## Worksheet 10



Can you learn to win this game?

## A Creative Mirror Reflection

### Activity with paint and paper folding

Providing an art experience for each child which will create a pattern in paint.

### Materials needed

Tempra paint (1 can of any color)

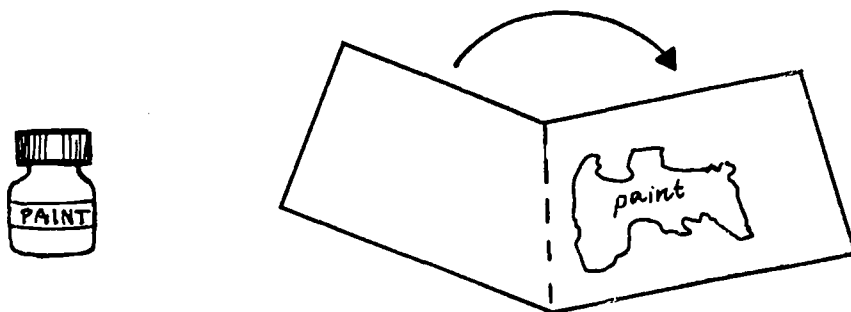
Paint brush

Provide for each child:

Crayons

8 x 12 sheet of white construction paper

### Background for the teacher



Instruct the children to fold an 8 x 12 white paper sheet in half. Open it and on one side have the children drop paint from a paint brush in one spot near the center of one half of the paper. See example above. The children should fold the paper over the paint and press it down.

When they open the paper two duplicate shapes will be seen each beyond the folded line which represents the line of symmetry. The painted area will usually emphasize the shape of some known object. If, for example, the painted area looks like the head of an animal, the children can add with crayon the body, legs and tail, etc. to make interesting pictures.

When completed each half of the total picture should be much like a mirror reflection of the opposite half.

## Letters with Lines of Symmetry

### Commentary for Worksheets 11 and 12

Discovering letters which have reflecting lines of symmetry.

#### Materials needed

Provide for each child:

pencil  
mirror

#### Background for the teacher

Ask the children to observe the letters on their worksheet.

Have them circle the letters which they think have reflecting lines of symmetry along the dotted lines.

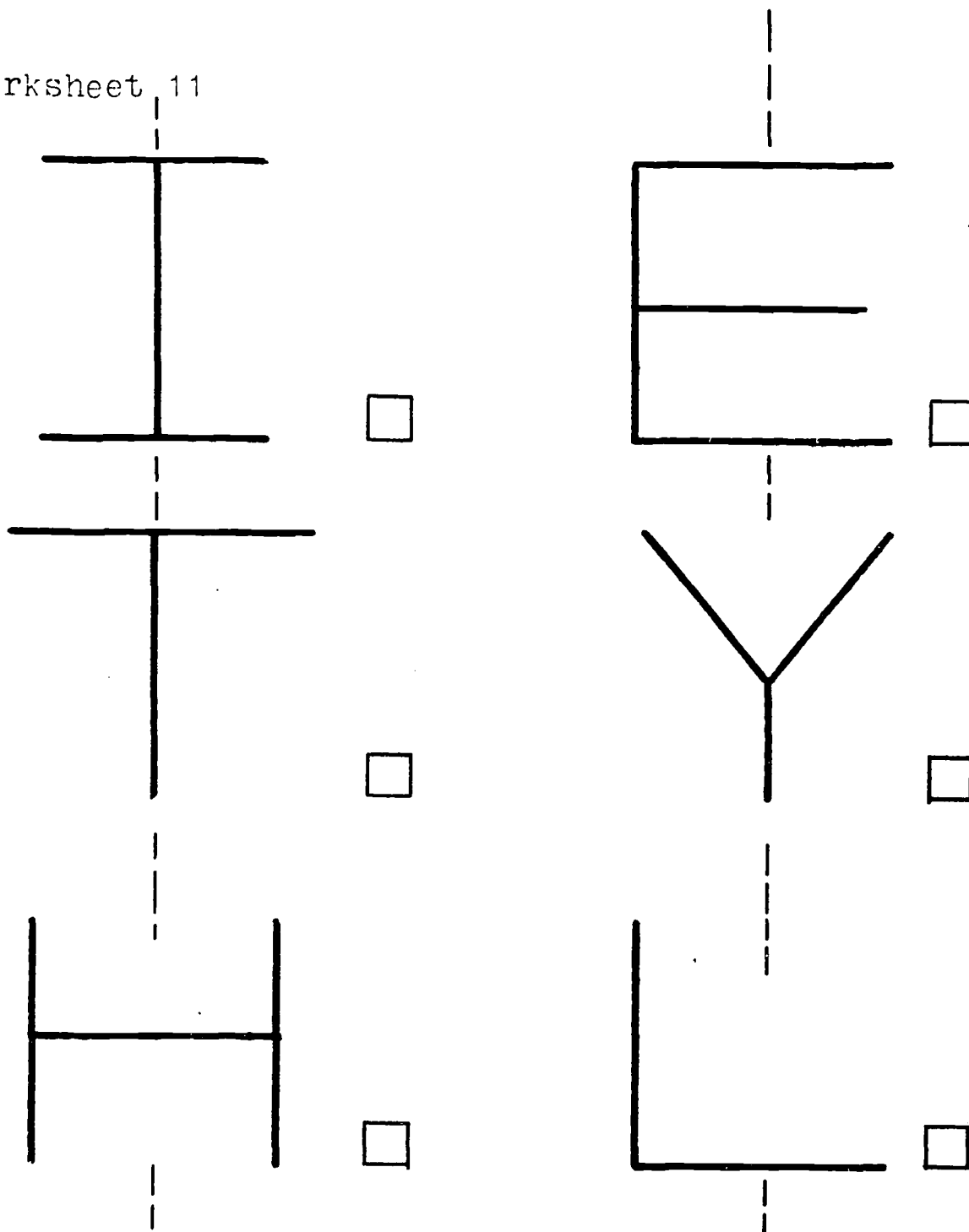
In the next step allow time for the children to use their mirrors to check if their "guesses" were correct. If they guessed correctly, ask them to put an x in the box next to each letter where they made the correct guess. If they made an incorrect guess, have them put an o in the box next to the letter.

Be sure that the children hold their mirrors along the lines of symmetry indicated by the dotted lines. The correct response will not occur if the mirrors are placed along the edge of the letters.

On a second activity which appears to look much like the first, have the children show which letters have vertical and horizontal lines of symmetry by marking those letters with red and blue crayons as directed on the worksheets.

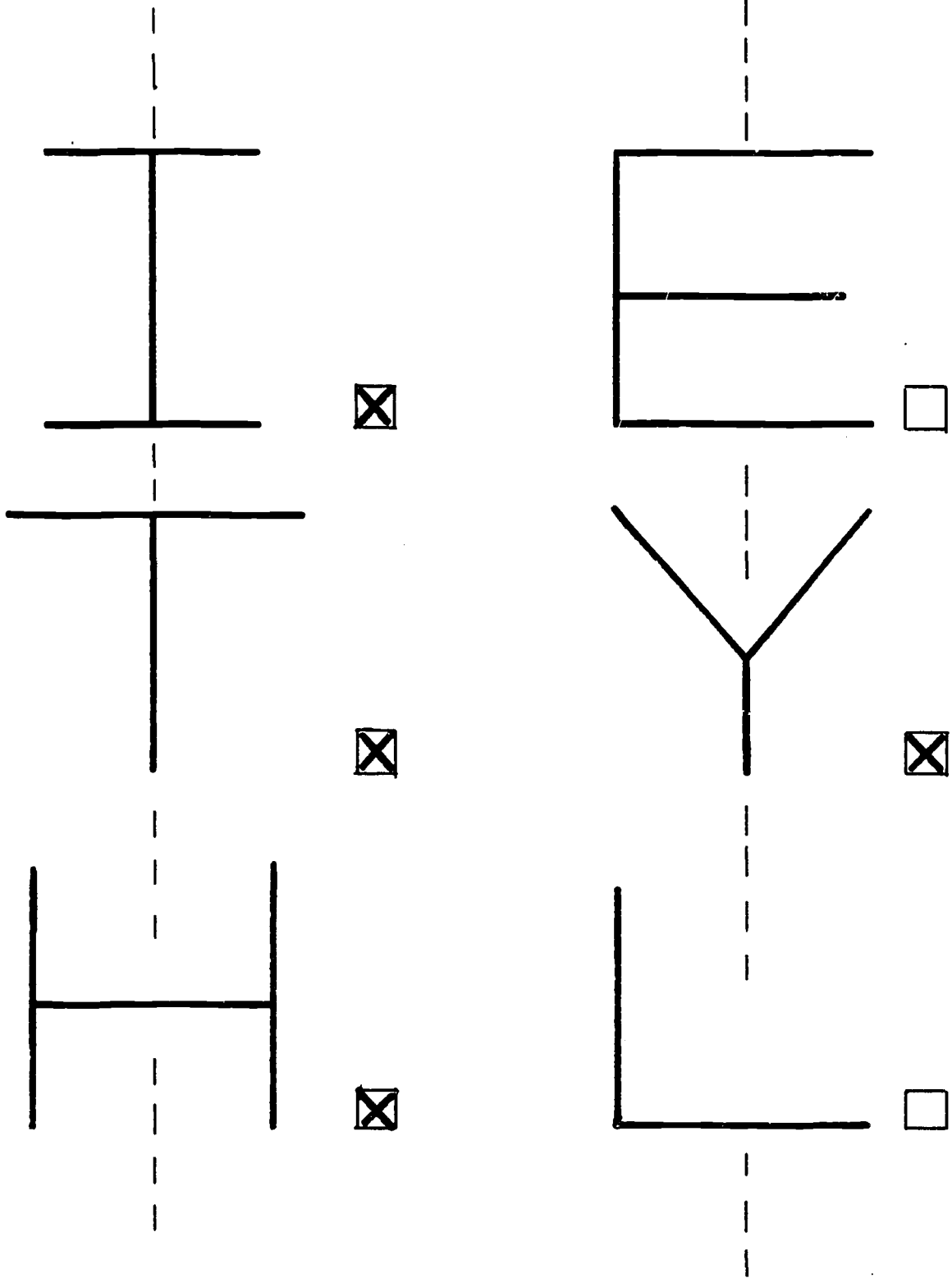


Worksheet 11

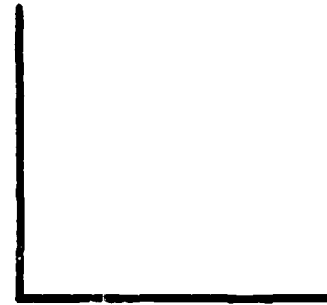
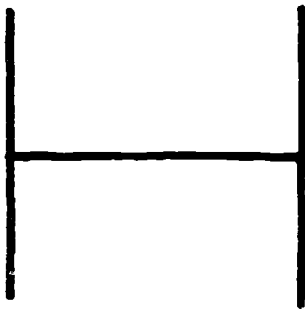
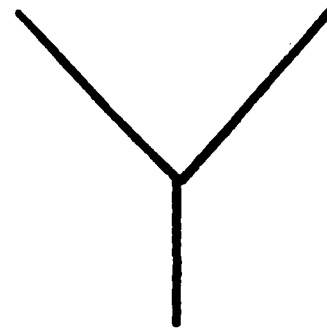
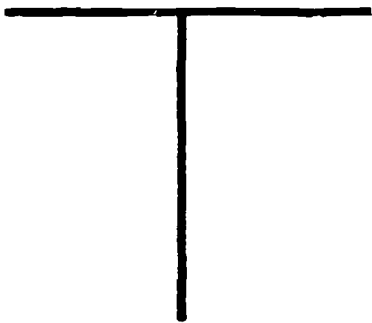
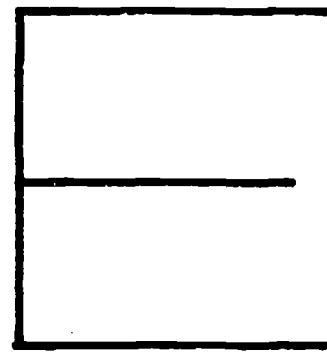
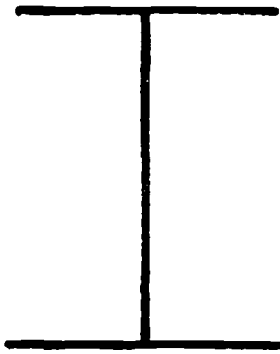


Which letters show mirror reflections? Be sure to hold your mirror along the indicated lines of symmetry.

Answers for Worksheet 11



## Worksheet 12



Show the letters that have a vertical line of symmetry by drawing a red line through that letter. Show the letters that have a horizontal line of symmetry by drawing a blue line through that letter.

## Words with Lines of Symmetry

### Commentary for Worksheet 13

Discovering words which have reflecting lines of symmetry.

### Materials needed

Provide for each child:

pencil

mirror

### Background for the teacher

Ask the children to observe the words on their worksheets.

Have them circle the words which they think have reflecting lines of symmetry along the dotted lines.

In the next step allow time for the children to use their mirrors to check if their "guesses" were correct. If they guessed correctly, ask them to put an x in the box to the right of each word where they made a correct guess. If they made an incorrect guess, have them mark the box with an o.

Worksheet 13

M O M

☐

D I D

☐

B O B

☐

D A D

☐

B I B

☐

Circle the words that you think have a mirror reflection along the line of symmetry. Then use your mirror. Put an X in the box for your correct guesses.

Answers for Worksheet 13

MOM



DID



BOB



DAD



BIB



## Finding a Corresponding Point

### Commentary for Worksheet 14

Finding a corresponding point in a pattern

#### Materials needed

Provide for each child:

pencil

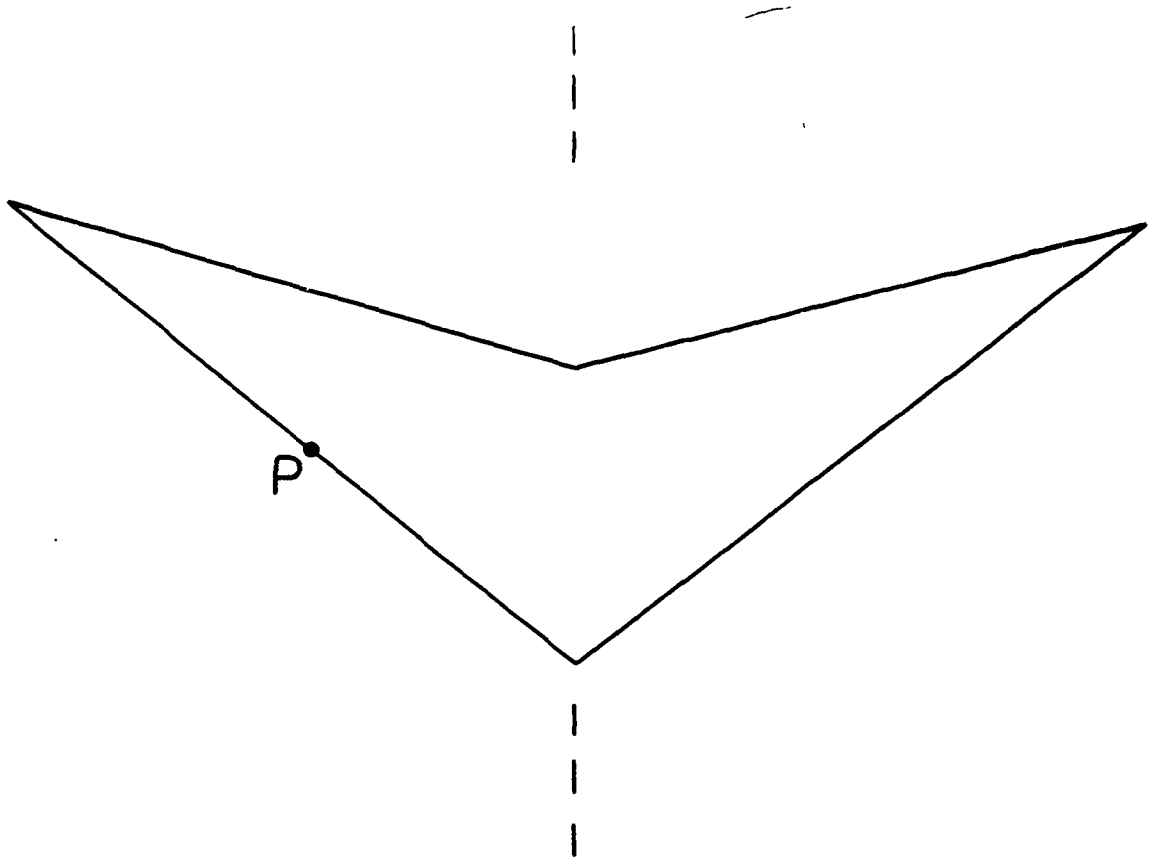
ruler

#### Background for the teacher

Ask the children to find the point  $p$  in the pattern. Direct them to mark the corresponding point as  $p'$ . Have them join  $p$  to  $p'$  by a line.

Complete the lesson by having the children fold their papers along the line of symmetry. Discuss with them what happens to line  $pp'$ .

Worksheet 14



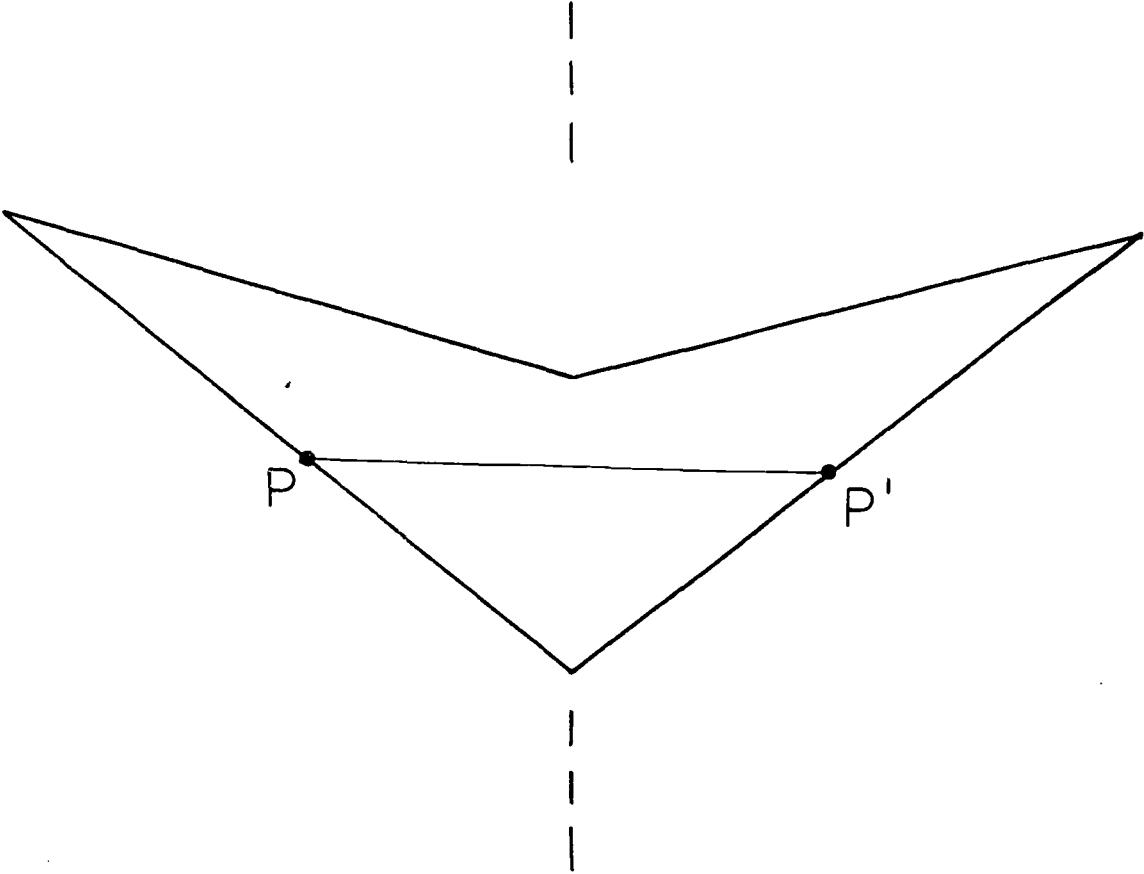
Find point p.

Draw a dot opposite point p on the pattern.

Fold the paper along the line of symmetry.

Hold the folded paper up to the light.





## Decomposing a Pattern

### Commentary for Worksheet 15

Observing that every point in the design is a mirror reflection of each corresponding point

Using tracing paper to record the various sectional parts

### Materials needed

Provide for each child:

ruler

pencil

three sheets of tracing paper

### Background for the teacher

Ask the children to study the design.

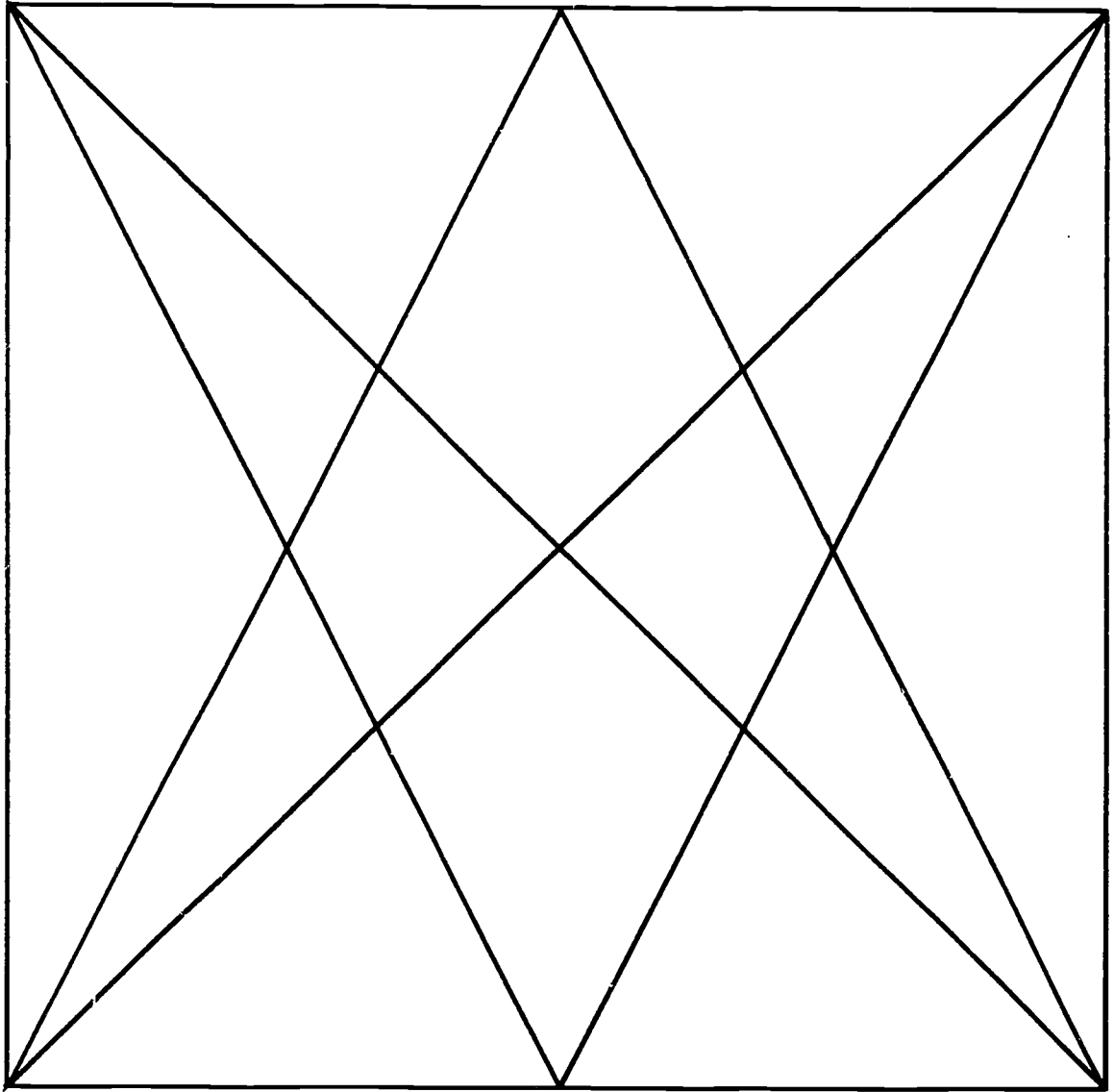
With the use of three sheets of tracing paper, ask the children to use the papers one at a time to trace a pattern from the original design.

Discuss with them that all parts of the pattern must be traced, but that no part may be traced on two sheets of paper.

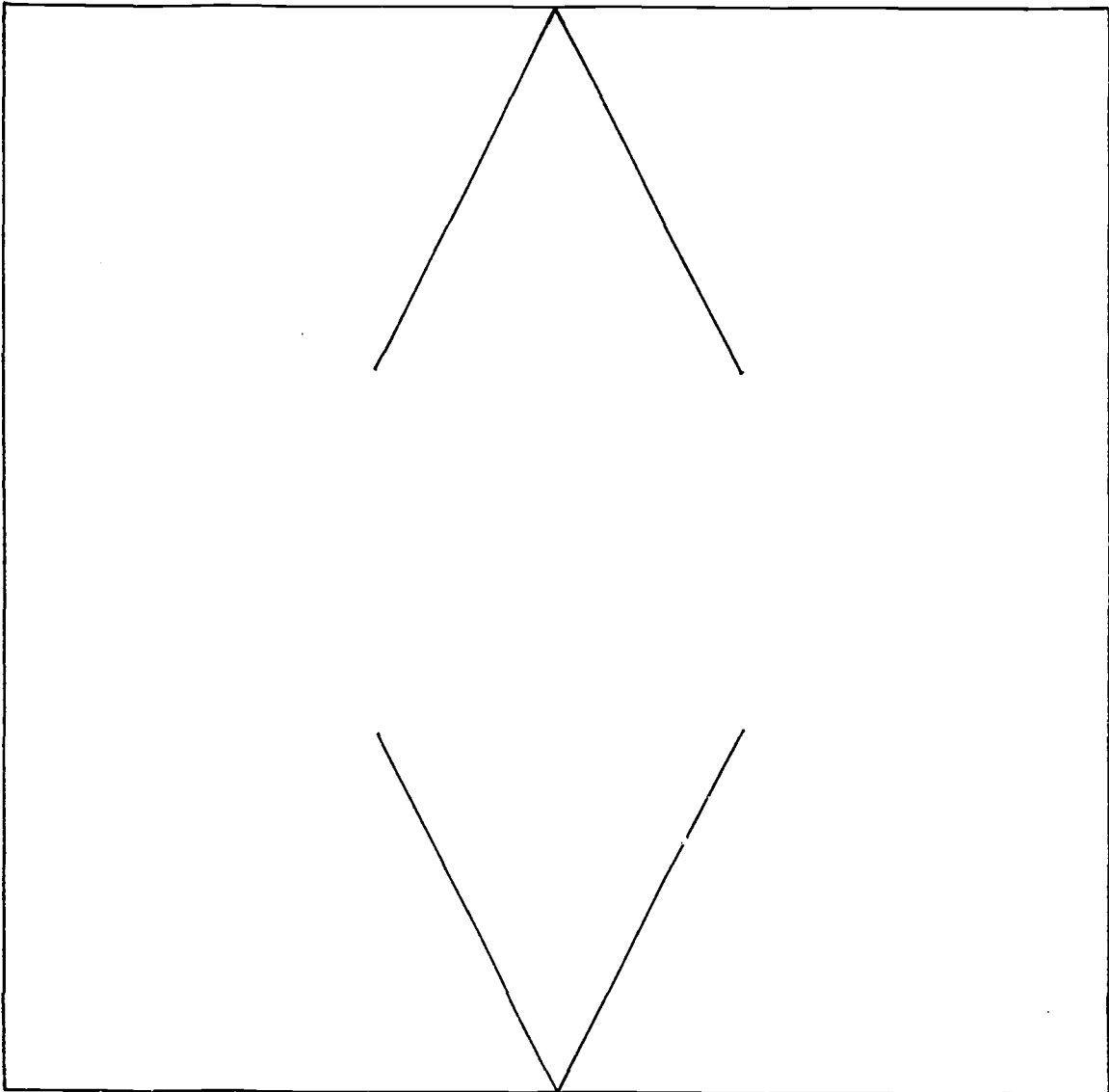
After the three mirror reflections have been drawn on the sheets of tracing paper, ask the children to place the three sheets together. Hold them to the light and they will see a duplicate of the original design given to them.

Three possible recordings follow in this teacher's manual. There are other possibilities. Some of the children's recordings will vary.

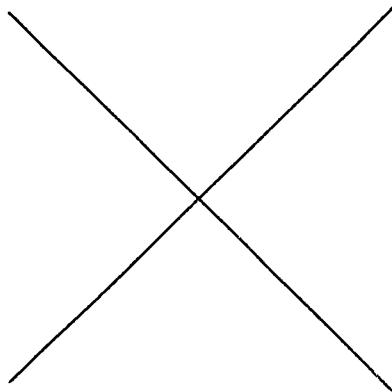
Worksheet 15



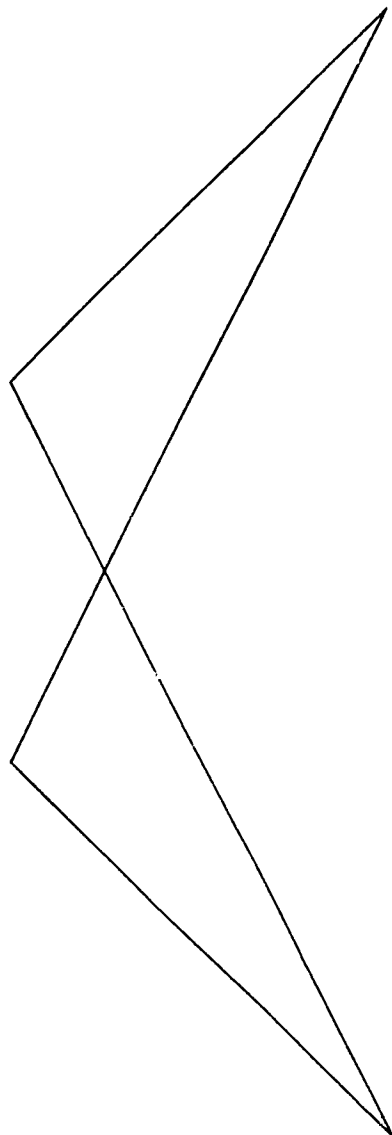
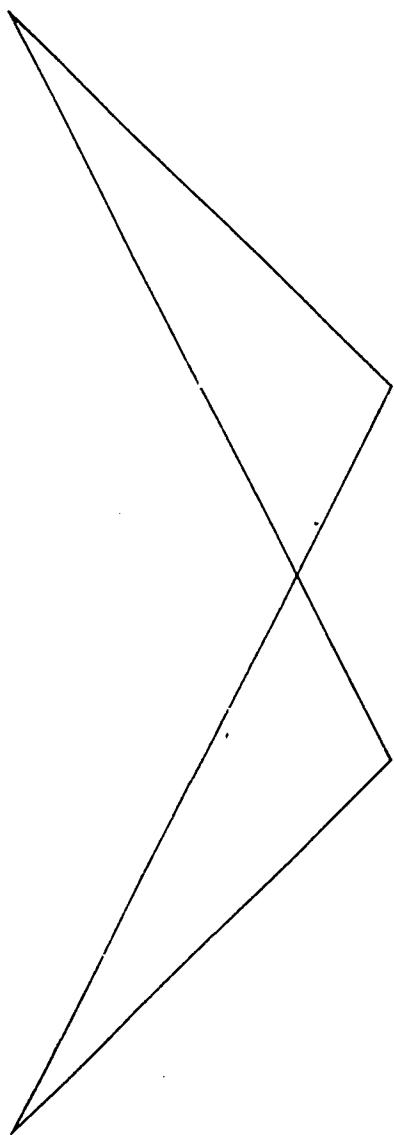
Answers for Worksheet 15



## Answers for Worksheet 15



# Answers for Worksheet 15



## ROTATIONAL SYMMETRY

### Rotation of Smaller Squares Around a Larger Square

#### Commentary for Worksheet 16

Discovering that the rotation of a pattern can make a mirror reflection of corresponding points  
Repeating a pattern in rotation

#### Materials needed

Provide for each child:

- pencil
- ruler
- mirror

#### Vocabulary

Introduce these words:

- rotate
- rotation
- clockwise

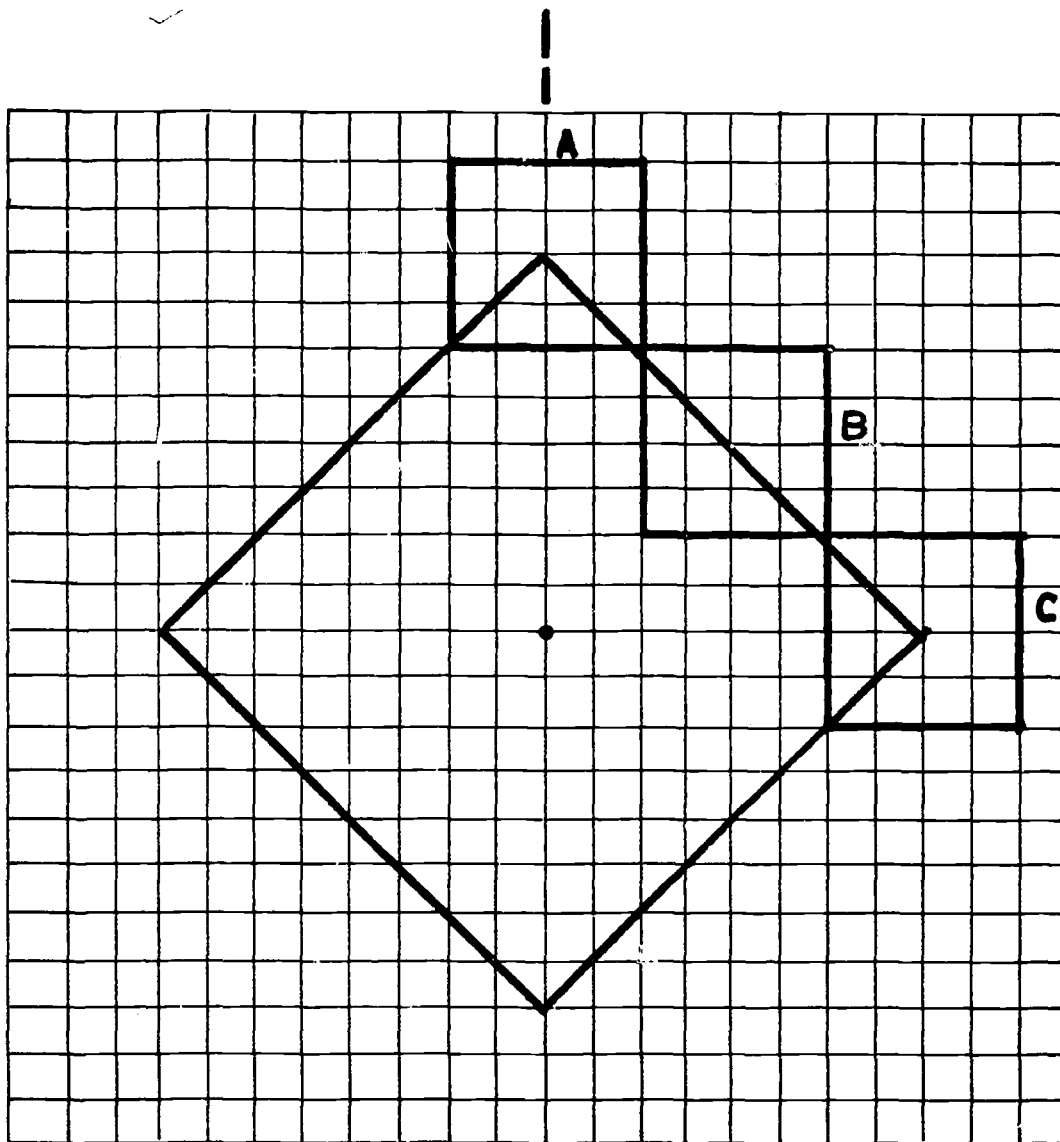
#### Background for the teacher

Give the children directions to study the drawings on their worksheets, ask them to draw in (rotate) small square areas of the same size as shown around the larger square areas until a design of symmetry is recorded. They should work clockwise around the square.

The squares labeled A, B and C are completed. As they draw in their squares, ask them to label the square areas with letters in order after C.

Ask the children to count the number of squares on their completed drawing. See how many children will also count the larger square.

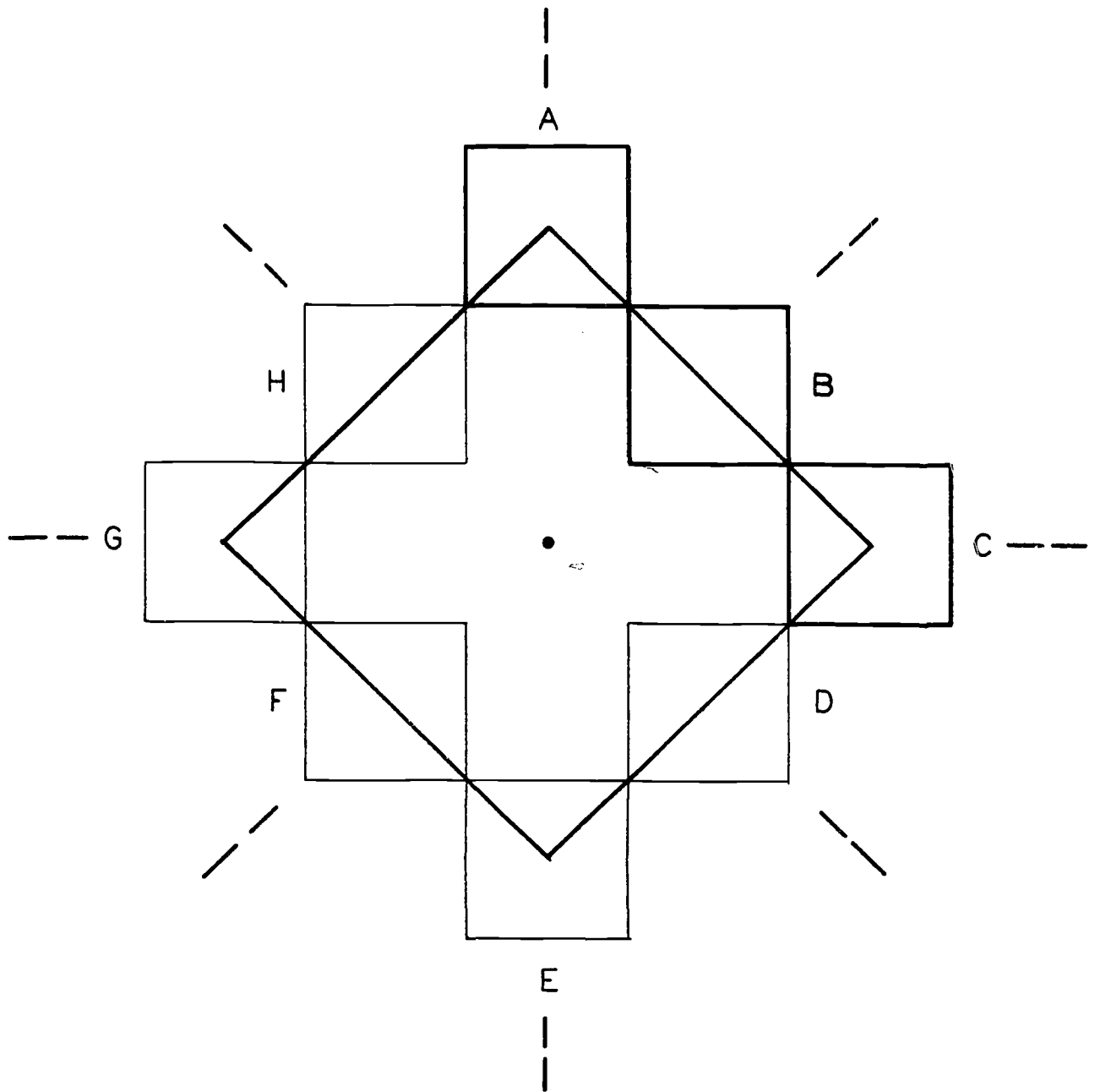
When the children complete their drawings (with the use of their rulers and mirrors), ask them to fold their drawings across the dotted lines and hold them up to the light. The children will discover that each side of their design is a duplicate of all other sides.



Continue drawing small squares in a clockwise way around the larger square. Label each square with the next letters of the alphabet. How many squares are there in all? Fold your paper along the lines of symmetry.



Answers to Worksheet 16



## More Rotation of Designs

### Commentary for Worksheet 17

Continuing the discovery that the rotation of a pattern can make a mirror reflection of the corresponding points  
Seeing that end points meet and can form a design

### Materials needed

Provide for each child:

- pencil
- ruler
- mirror

### Background for the teacher

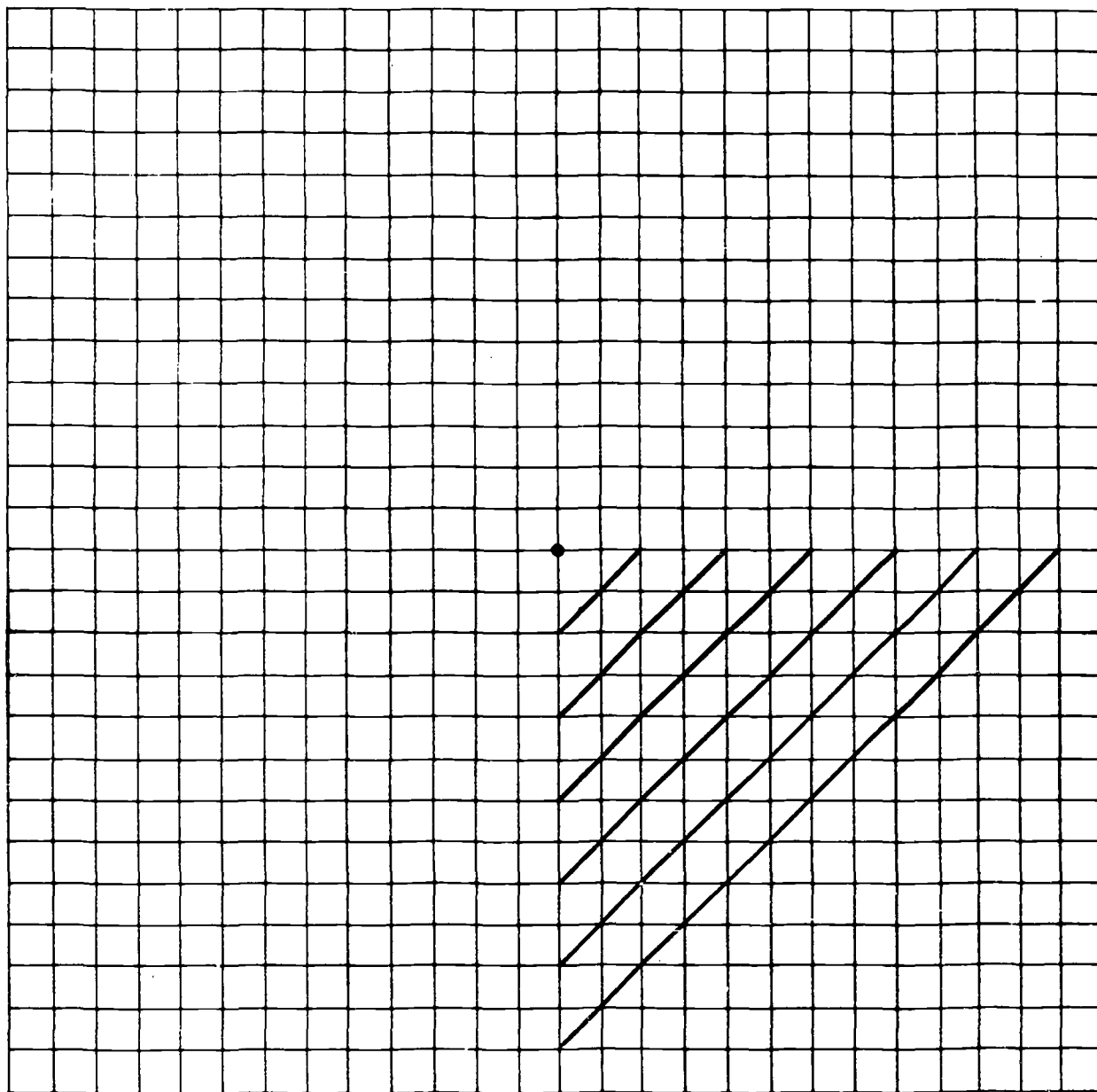
Have the children study this design. Then, using their rulers and mirrors, ask them to finish the pattern by drawing the lines in the remaining three sections.

After the children have completed the pattern, have them put their finger in the center of the paper and rotate the paper around until the pattern is the same as before the rotation.

Ask the children, "to how many positions can you rotate the page and still have the pattern the same?" (The answer is four.)

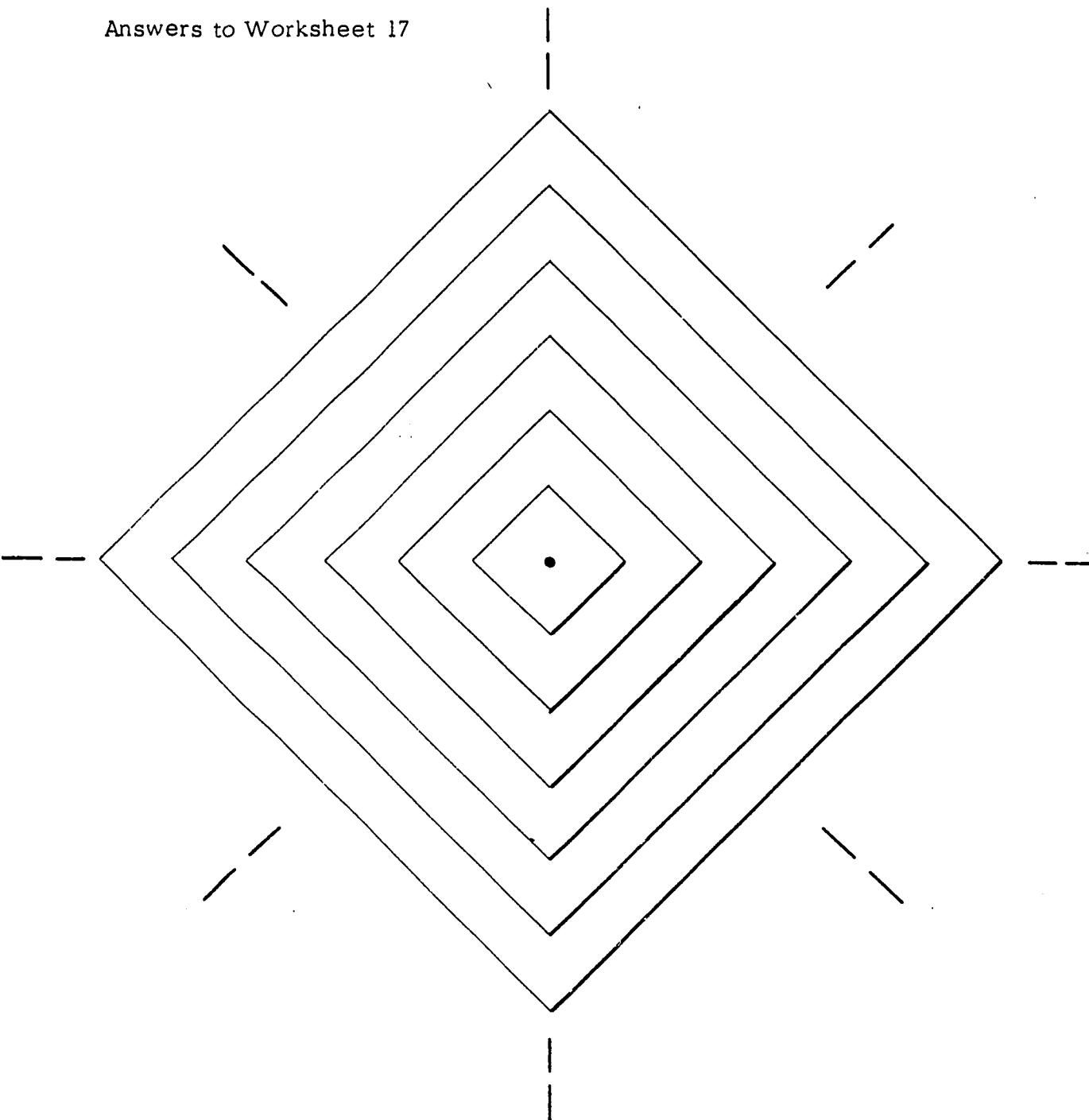
The children who enjoy this type of lesson will want to create their own patterns of rotation.

Quarter-inch graph paper should be made available for the children who choose to participate in a continuing activity.



One fourth of this design is drawn. Finish the design. Fold along the lines of symmetry. After folding each part, hold it to the light. Do the points fall over each other?

Answers to Worksheet 17



## Shifting Patterns in Symmetry

### Commentary for Worksheet 18

Discovering that the shading in of alternate regions results in a shift in the pattern of symmetry.

### Materials needed

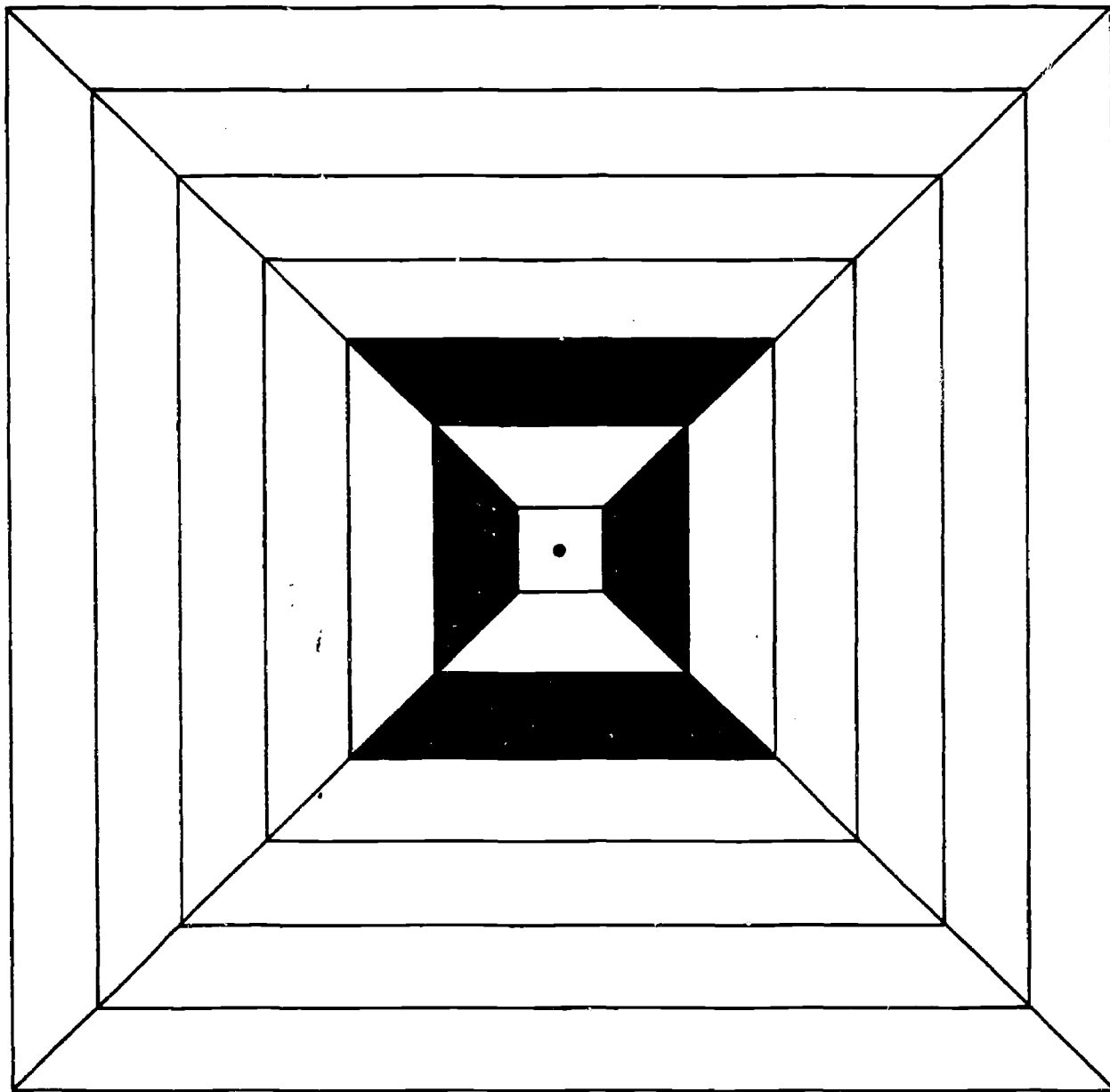
Provide for each child:  
a pencil

### Background for the teacher

Four regions are shaded in on the children's worksheet. Ask the children to continue shading in alternate regions of their design until they have reached the outer edge. Caution the children that the outer edge may not be filled in all the way around.

When the designs have been completed, have the children rotate the paper as before, to see the rotational symmetry. (This time, only two positions give the same pattern.)

Besides rotational symmetry, this pattern has bilateral symmetry. Ask the children to fold their paper on the dotted line and then hold it up to a light so that they can see this.

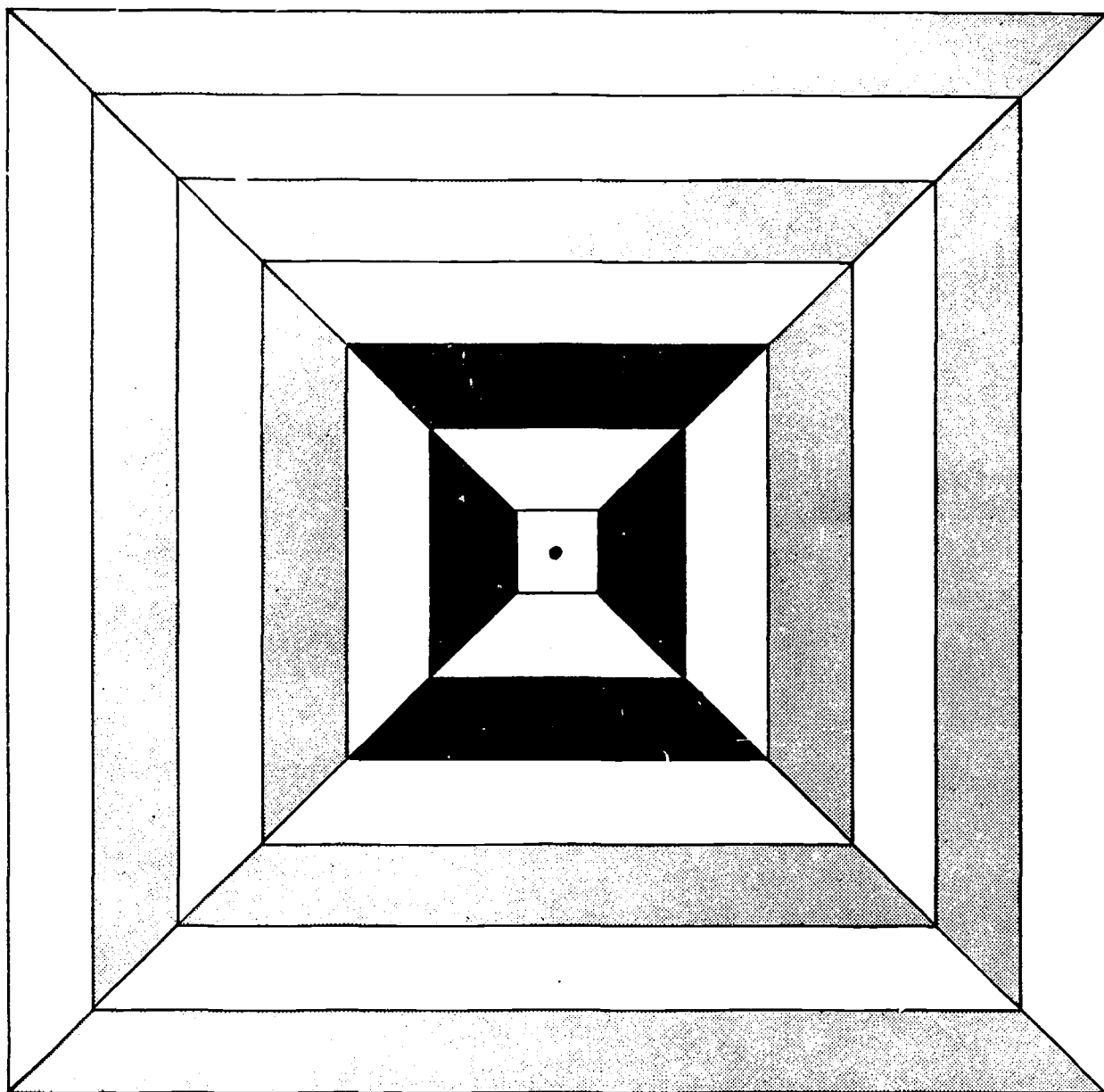


Leave a space unshaded. |

Shade in the next part.

You are shifting patterns of symmetry.

Answers to Worksheet 18



## Exaggerating the Size of a Square

### Commentary for Worksheet 19

Helping children learn to enlarge squares from a smaller square  
Observing the continuing pattern of symmetry that squares make

### Materials needed

Provide for each child:

ruler  
pencil

### Vocabulary

Introduce these words:

increase  
decrease

### Background for the teacher

Direct the children to look at the small  $1/4$ " size square at the lower left hand corner. Ask them to skip one space above and then draw a square two units ( or  $1/2$ " size) on the graph paper.

Encourage them to continue making squares each of which increase one unit (or  $1/4$ " ) in size until they have made a square seven units or  $1\ 3/4$ " size square.

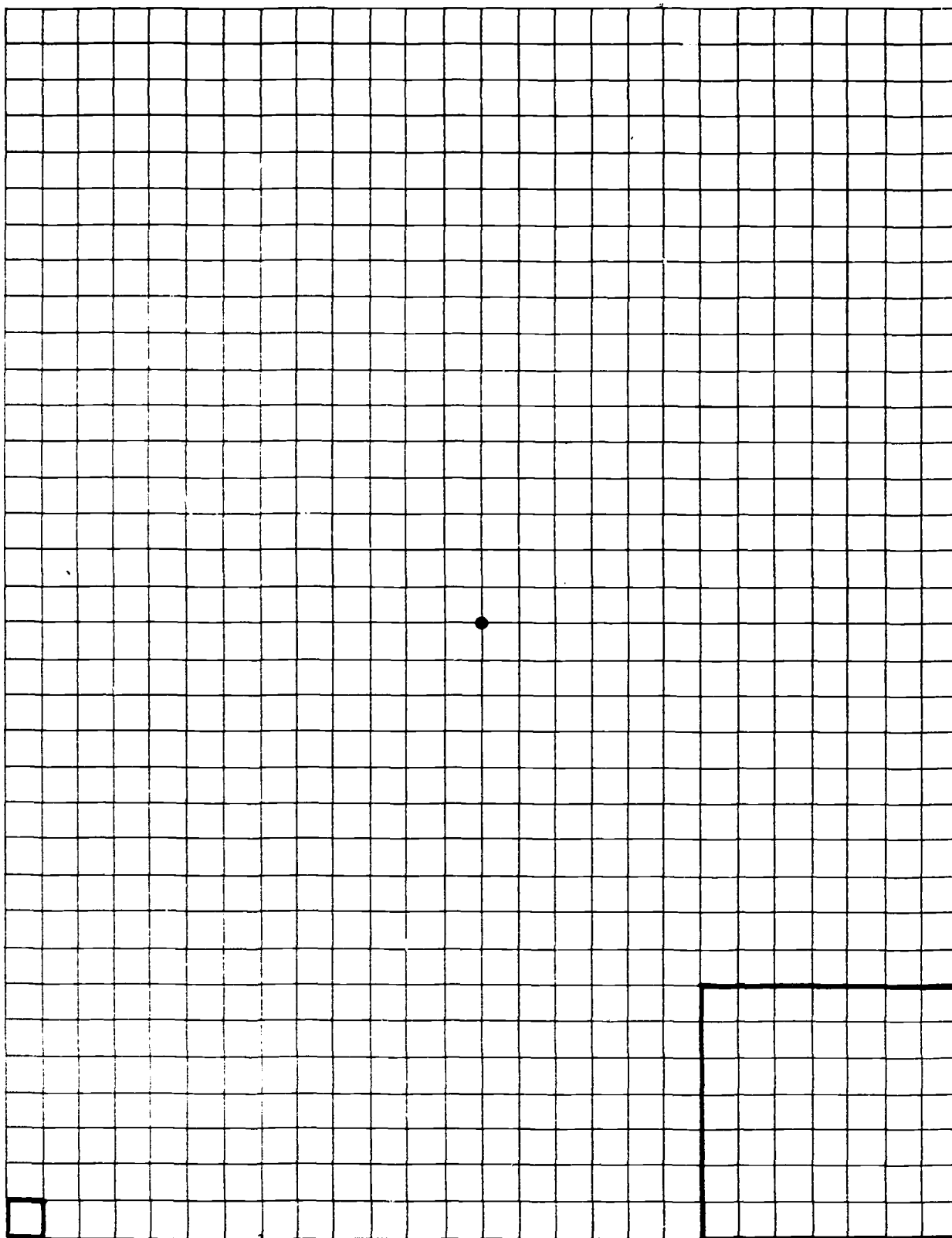
The children should understand that a space must be skipped between each square and that each square should begin in line with the previous one.

After exaggerating the first row of squares ask the children to find the large size square at the bottom right hand corner and to decrease each square one-quarter inch in size (always skipping a space) until a square one-quarter inch in size will be drawn near the top of the paper.

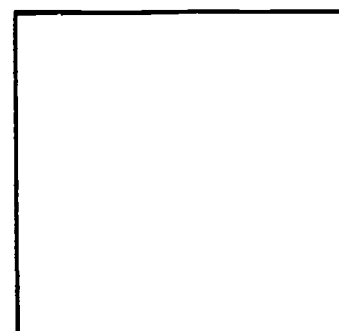
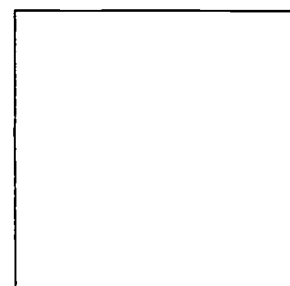
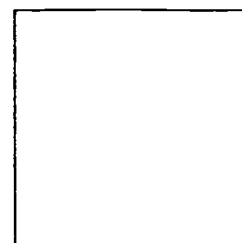
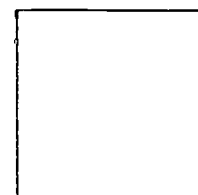
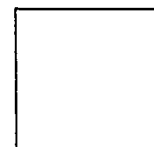
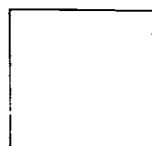
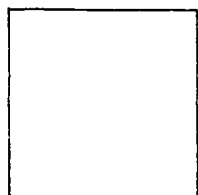
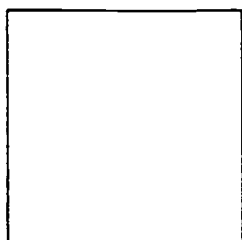
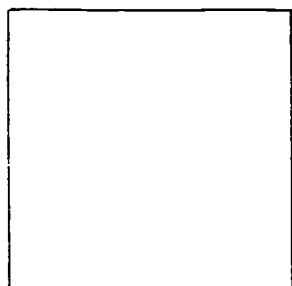
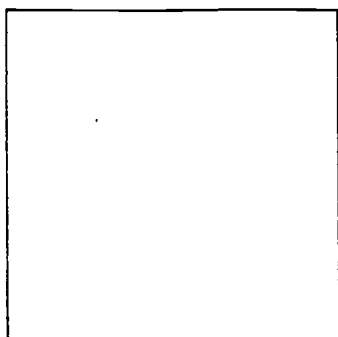
Instruct the children to rotate their paper as before, to see the rotational symmetry. (The pattern is the same for two different positions of the paper.)

Ask the children if there is any way that they can fold their paper to show bilateral symmetry. Let them experiment with their paper even though no solution will be found.





# Answers to Worksheet 19



## TRANSLATORY SYMMETRY

### Drawing a Parallelogram

#### Commentary for Worksheet 20

Becoming acquainted with the parallelogram

#### Materials needed

Provide for each child:

pencil

ruler

sheet of tracing paper

#### Vocabulary

Introduce this word:

parallelogram

#### Background for the teacher

Direct each child to place a sheet of tracing paper over the top of his work-sheet. Have the children put a dot over  $P$ ,  $P'$ , and  $P''$ .

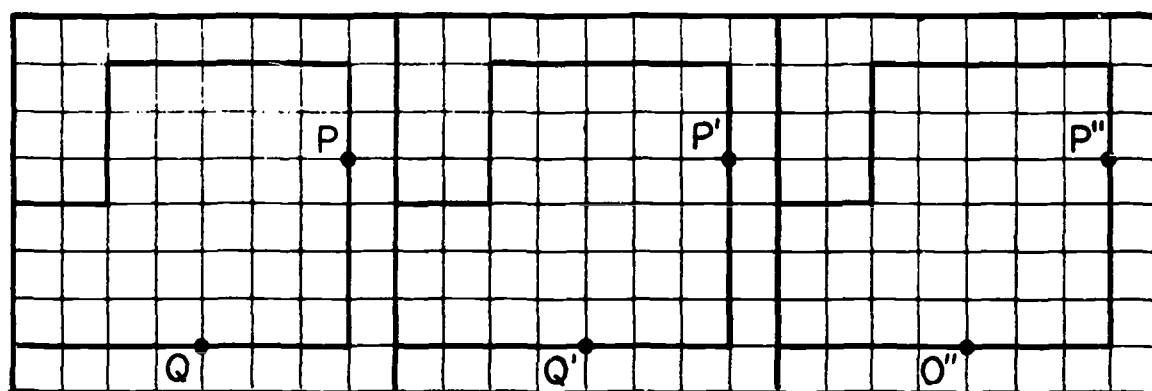
As the lesson continues have the class join (on their tracing paper sheets)  $P$  to  $P'$  by a straight line, and  $P'$  to  $P''$ . What do the children notice? How do the distances  $PP'$ ,  $P'P''$  compare?

Repeat with point  $Q$ . How do the distances  $QQ'$ ,  $Q'Q''$  compare?

Extend lines  $PP'$  and  $QQ'$  in both directions. Do they meet, and if so where? Do the same with  $PQ$  and  $P'Q'$ .

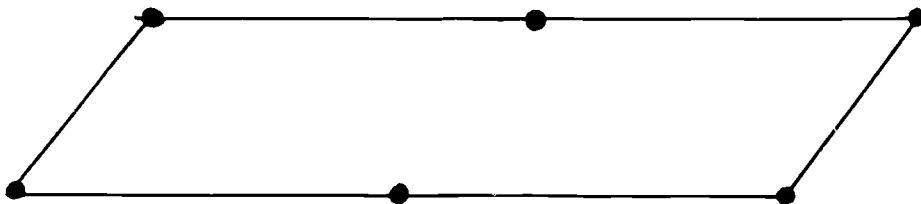
The figure which will be recorded on the tracing paper sheet will be a parallelogram.

# Worksheet 20



Use a sheet of tracing paper. Put dots on the tracing paper over the six dots on this paper. Connect the dots as directed by your teacher. What geometric shape do you see?

## Answers to Worksheet 20



## A Creative Border in Balance

### Commentary for Worksheet 21

Giving children an opportunity to create a border with a repeating pattern

### Materials needed

Provide for each child:  
crayons (or colored pencils if available)

### Background for the teacher

Ask the children to make an original pattern in the first square of the border. In each of the other four square areas instruct them to make a repeating pattern of the original.

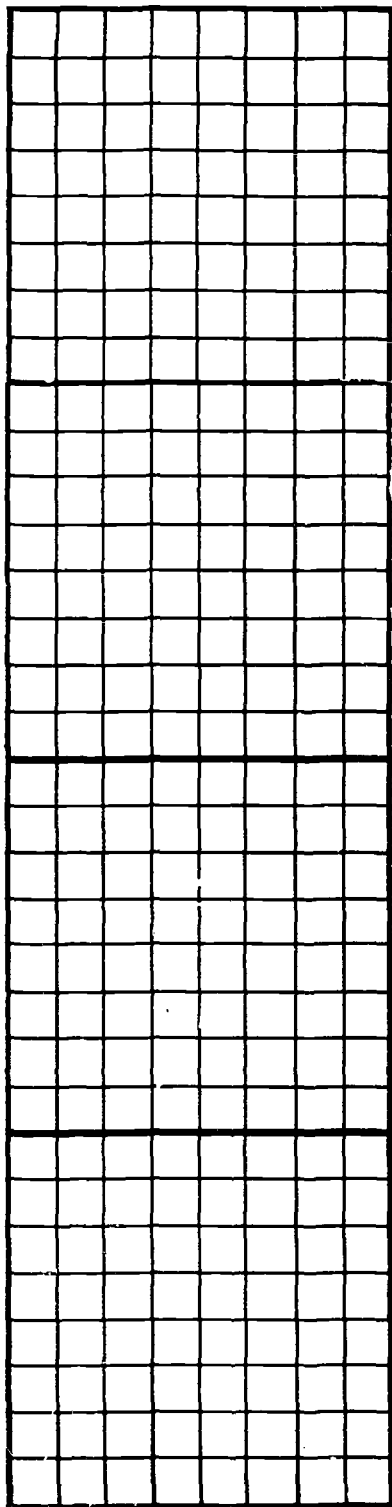
Explain to them that this is how people design head bands, belts, wall-paper border, etc.

This experience will give children an opportunity to use their creative talents in designing and to observe patterns that balance one with the other.

It is recommended that the designing be done with crayons although colored pencils may be used if they are available. Children can also make interesting borders by shading in the various areas with black lead pencils.

It is common for children to want to shade or color in every other square which makes for a checkerboard affect. Mention could be made to them that interesting patterns can be made by many arrangements of shading or coloring.

## Worksheet 21



Make a pattern in  
the first square.  
Make your pattern  
the same in the other  
squares. When you  
are finished you will  
have a border.

## Sing a Song of Symmetry

"Sing a Song of Symmetry" is an example of translatory symmetry in sound. Writing the notes of the repeating phrases side by side will give a visual reinforcement to the perception of a repeated phrase in sound.

The rhythmic accompaniment will be learned separately by imitation and repetition. The whole class may join in learning the rhythms by stamping with the tambourine and clapping with the wood-block, thus

stamp, clap, stamp, clap, stamp, clap-clap-clap, stamp

The ending, "Pick it up, etc." could alternate voices with instruments and claps, and with a bang at the end as if someone slammed the lid of the phonograph.

Pick it up/clap-clap-clap/Pick it up/clap-clap-clap/ - wait - BANG!



# SING A SONG OF SYMMETRY

PRECISELY

Voices

PAT - TERNS KEEP RE - PEAT - ING.  
 DRUM - STICKS KEEP ON BEAT - ING.

WOOD-BLOCK

TAMBOURINE

LES - SONS NEED COM - PLET - ING.  
 CHIL - DREN KEEP ON EAT - ING.

SOUNDS LIKE THE PHON-O - GRAPH NEED-LE'S STUCK. PICK IT

UP ! PICK IT UP ! PICK IT UP ! PICK IT UP ! PICK IT - ETC.